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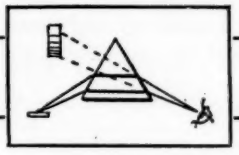

NOVEMBER
1933

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Line Shape as a Function of the Mode of Spectrograph Slit Irradiation

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(Received July 3, 1933)

Some of the more common modes of spectrograph slit irradiation have been studied experimentally. Particular attention has been paid to line shape and its effect on resolution. The noncoherent mode is seldom if ever employed because of practical limitations. The lens and broad source modes offer a means of realizing high flux transmission, but the consequent line resolution may be poor. The resolution can be improved by reducing either the relative aperture of the condenser lens or the angle subtended by the broad source at the slit. The limit in either case is the coherent mode with which very sharp, well-resolved lines can be obtained. The closest approach to flat top line shape is realized through the use of a wide slit when the departure from noncoherent irradiation is not very great.

IN an earlier note[1] mention was made of the studies which we have been making of various modes of spectrograph slit irradiation. These studies should be useful since the literature indicates that most spectroscopists give too much attention to the quality or reputation of their instruments and too little to the manner in which they are employed. The way in which radiation is made to pass through a spectrograph has so great an influence on the results that sometimes the mode of irradiation is more important than the quality of the optical system. This paper summarizes some of the practical relationships between mode of spectrograph slit irradiation and spectral line shape, and indicates how the latter affects resolution. For a theoretical discussion of the general problem see van Cittert's paper.[2]

Although this summary was prepared primarily with the spectrograph in mind, the principles are applicable to any optical instrument possessing a slit and collimator, e.g., the spectroscope and the spectral radiometer. Since the final results are influenced in any case by the characteristics of the detector, it is to be expected that the conclusions reached in the following sections must be modified somewhat before they can be applied directly to the consideration of other instruments.

RESOLVING POWER

The resolving power of a spectroscope is defined as the ratio of the mean wave-length of a pair of lines which can just be "resolved" to the difference in wave-length between them. Evidently the criterion of resolution is merely that the two lines shall be separated. Since the necessary degree of separation depends on whether the lines are to be detected visually, electrically, or photographically, and again on whether photographic records are to be examined

[1] D. C. Stockbarger and L. Burns, Phys. Rev. **37**, 920 (1931).

[2] P. H. van Cittert, Zeits. f. Physik **65**, 547 (1930).

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visually or microphotometrically, etc., it follows that resolving power is not to be regarded as a constant. Furthermore a resolving power satisfactory for wave-length determinations may be entirely inadequate for energy measurements, for in the former only the positions of peaks need be discerned, whereas in the latter partial overlapping throws energy from one line into another. The overlapping of peaks is generally disastrous, but a serious error can result even from adding the energies of secondary maxima of neighboring lines to the central energy of the one being measured. It is therefore not sufficient, in general, to limit the half-widths of lines; the background of secondary maxima needs to be eliminated as far as possible, a matter which may easily be overlooked in discussions of resolving power.

In wave-length measurements, also, secondary maxima of neighboring lines can be sources of error. Furthermore, the peak being the point of highest intensity, apparent wave-length shifts sometimes result from a change in energy distribution within the line, i.e., a change in line shape, the resolution remaining practically constant. Such shifts amounting to several Angstroms have been noted in the course of our experiments.

While the term resolution will be used freely, no attempt will be made to evaluate the quantity in terms of theoretical resolving power for the latter evidently has lost most or all of its former significance. Such an evaluation would necessitate establishing an arbitrary standard of line separation and would therefore be of limited use.

Slit Widths

Theory and experiment show that for every mode of spectrograph slit irradiation there is one slit width which is best from the standpoints of line intensity and resolution. If the slit be made wider than the critical value there results a loss of resolution with no gain in intensity. If it be made more narrow there is a serious loss in intensity with little or no gain in resolution. Indeed, if the slit be made too narrow there may be a loss of resolution due to the appearance of satellites. The numerical value of the critical slit width, measured always in wave-lengths, depends on the mode of irradiation and cannot be predicted exactly except in limiting cases. It can be found experimentally, however, and should be thus determined in any case. Unless otherwise indicated, the critical slit width was employed in each of the illustrative examples presented in this paper.

Noncoherent Mode

The noncoherent mode, assumed in most discussions of resolution, is a natural goal of spectroscopic workers for with it maximum flux transmission is realized. By definition, rays from every part of the slit reach every part of the collimator surface in a perfect mixture of phases. This calls for a self-luminous slit or really no slit at all but a narrow source placed at the focus of the collimator, as indicated by Fig. 1. Diffraction, the phenomenon which limits resolution by introducing secondary maxima, can come only from the lenses and prism or from the grating. Since this is reduced by increasing the aperture, very nearly perfect lines should be obtained with an instrument of very large aperture if it were possible to make the source sufficiently narrow. Satisfactory line radiation sources of slit dimensions are not obtainable, however; and the requirement is only made more difficult by increasing the aperture. A simple calculation will make this clearer. For the noncoherent mode the best width of self-luminous slit is equal to the wave-length multiplied by the ratio of focal length to diameter of the collimator. For the wave-length, 4000A, and an aperture ratio of 10, the width is found to be 0.004 mm. This mode need not be considered further in this paper except as an ideal case from which departure is made necessary by experimental difficulties.

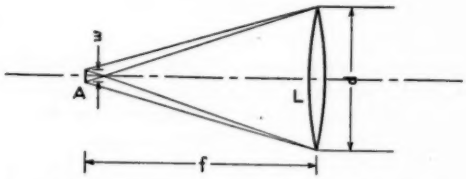

Fig. 1. Diagrammatic representation of the noncoherent mode. Source A of infinitesimal thickness and width w placed in the focal plane and on the optical axis of collimator lens L having a diameter d and focal length f. $w=\lambda f/d$.

The most common methods of approaching the noncoherent mode are to place a broad

source before the slit (broad source mode) and to focus an image of the source on the slit (lens mode). Very nearly maximum flux transmission is realized in either way, but additional diffraction appears as a result of the radiation passing through, rather than originating at the slit. This defect is a characteristic of all practical modes and often is the cause of serious loss of resolution, appearance of satellites, line multiplicity, and other line deformities. As will be shown below, the line shape can be improved greatly by decreasing the angle subtended by the source at the slit so that less than maximum flux is transmitted. When this is carried to the limit, the coherent mode is reached.

Coherent Mode

We define coherent irradiation as that mode in which all waves reach the slit in the same phase and speak of the resulting spectral lines as "coherent lines." While this case can be realized in theory only through a point source at an infinite distance from the slit, in practice we need only place a one millimeter wide source, e.g., a narrow discharge tube, at a distance of two meters from the slit of a Hilger E2 spectrograph to obtain typical coherent lines. This is illustrated by Fig. 2. Even with somewhat broader sources, or shorter distances, the departure from coherence may not be great. It should be clear that the horizontal angle subtended by the source at the slit is the chief determing factor and not the relative size of the source alone. A lens placed between the source and slit could destroy the coherence regardless of the minuteness or distance of the source.

Perhaps the most outstanding characteristic of the coherent mode is the high resolution which it is capable of yielding. Indeed, as van Cittert[2] has indicated, the ideal noncoherent mode should not yield much narrower lines. The lines are very sharply defined, provided that the slit is not too narrow, and remain so even when the slit is made so wide that they overlap. This fact was illustrated in our earlier note. Somewhat finer central lines can be obtained through the use of slits narrower than the critical value but they are less desirable from the standpoint of sharpness and they unfortunately require longer exposures. (See Fig. 3.) Although slits wider than the critical value still yield sharp lines, such slits will not ordinarily be adopted because of the resulting multiplicity. Double lines appear when the slit width is twice the critical value, and so on.

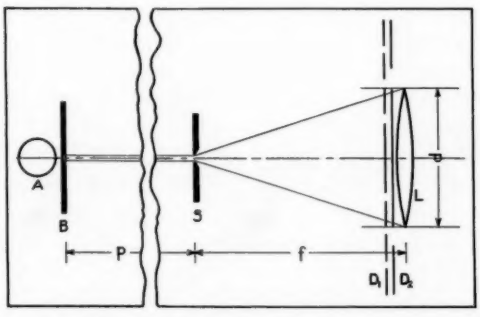

Fig. 2. Diagrammatic representation of the coherent mode. Source A screened by a baffle B having a small opening (order of 1 mm) placed on the optical axis of slit S and collimator lens L of a spectrograph. The distance p from baffle to slit should be several meters. With the slit critically adjusted for radiation of wave-length λ the diffraction pattern as seen by the eye, placed approximately in the focal plane of the spectrograph, is shown diagrammatically by D_2. It will be noted that the lens is filled by the central maximum. Halving the wave-length or doubling the slit width causes the diffraction pattern to take on the appearance indicated by D_1. Note that the lens diameter must now be reduced to $d/2$ in order to include only the central maximum.

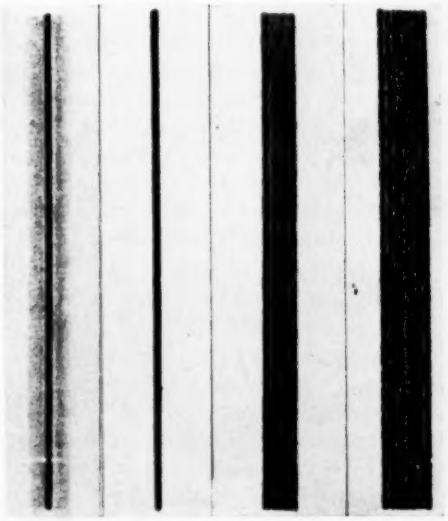

Fig. 3. Hg λ4078 photographed by the coherent mode with several different values of slit width, *viz.*, (a) less than, (b) equal to, (c) five times and (d) eight times the critical value. Note the pronounced rim effect in the multiple lines. Hilger E2 spectrograph used throughout the work reported in this paper.

As produced by ordinary exposures, such as would be adopted for quantitative work, coherent lines made with the critical slit width are isolated and sharp. Indeed their appearance suggests their having been drawn by a fine ruling pen. Very faint secondary maxima sometimes appear as a result of prolonged exposure, but these do no harm in practice for by the time they reach objectionable magnitude the parent lines are already too dense for intensity measurements and too broad from overexposure to be suitable for accurate wave-length determinations. A further distinction is that the secondary maxima, or satellites, are also isolated and sharp instead of being more or less fused with the central line. When only approximately coherent slit irradiation is employed, the central lines are somewhat broadened, the satellites appear even in practical exposure times, and the general isolation of maxima is less evident. Nevertheless. such lines may be fairly sharp and hence useful, They possess the advantage of requiring shorter exposures than do truly coherent lines.

Because of the small amount of flux transmitted, requiring either bright sources or long exposures, it is not to be expected that the coherent mode of slit irradiation will ever compete for popularity with other modes. It is not recommended for general adoption but rather as a limit toward which spectroscopists should work to whatever extent their individual resolution requirements may indicate. Certainly, however, it should be given serious consideration whenever spectrograms are being prepared for reproduction in journals or lantern slides.

Lens Mode

In some respects a close approach to the noncoherent mode may be realized when an image of the source is focussed on the slit, as indicated by Fig. 4. If the source had negligible depth, the

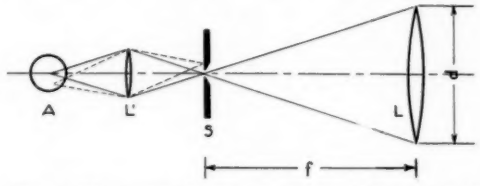

Fig. 4. Diagrammatic representation of lens mode. An image of some part of source A is focussed by lens L' on the spectrograph slit. Other parts of the source are focussed elsewhere.

condenser lens were very large and optically perfect, and the image were of slit dimensions and in the slit plane, we might expect good line shape. Since all of these requirements cannot be met, there is in effect a series of sources located both before and behind the slit, i.e., there is a departure from the conditions specified by definition for the noncoherent mode in that not all of the flux originates in the slit plane. The consequence is line broadening, which cannot be controlled much through adjustment of slit width since the radiation does not arrive at the slit as a plane wave.

Coherent irradiation being approached as the diameter of the condenser lens is reduced, it is to be expected that an improvement in resolution should result from reduction of the relative aperture and simultaneous slit width adjustment. This is verified by the experimental results shown in Fig. 5. Evidently the relative aperture

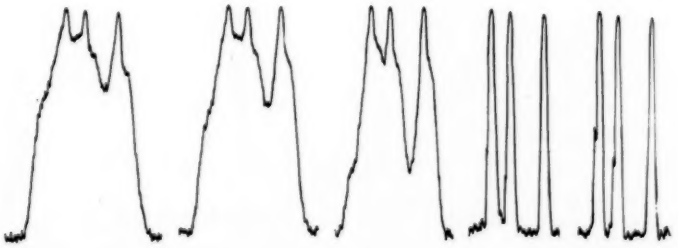

Fig. 5. Photometrically recorded spectrograms of Hg λλ3650–63 made with an image of the source focussed on the slit of the spectrograph illustrating the improvement in resolution resulting from reducing the diameter of the condenser lens. The relative aperture of the condenser lens was 1. 0.8, 0.5, 0.3 and 0.06, respectively, referred to the collimator.

of the condenser lens should be no greater than that required to insure records being obtained in a reasonable exposure time. This mode possesses the advantage that as great a degree of collimator filling as may be desirable can be obtained even with a point source, so that high flux transmission is always possible, but this advantage is more or less offset by a number of inherent defects.

Assuming chromatic defects in the condenser lens to have been eliminated, or better that observations are to be made within a narrow spectral region, there remains the fact that source defects are projected on the slit and hence may be introduced in the line. In effect, the line records the radiation of some particular wavelength from a more or less well-defined slice of the source cut in a plane parallel to the slit and to the axis of the collimator. Unless the source be uniform in energy distribution, the observed line intensity cannot be representative of the source as a whole. There are cases, of course, in which this characteristic can be put to practical use, as when it is desired to study radiation originating in some specific region of a discharge; but in general any mode which allows the source geometry to affect the line so strongly, needs to be employed with caution. Some altogether strange line shapes have been known to result from the projection of small imperfections in the glass of a discharge tube.

Broad Source Mode

The broad source offers the simplest means of approaching the noncoherent mode. As indicated by the sketches in Figs. 6 and 7, the source is placed in line with the collimator axis and at

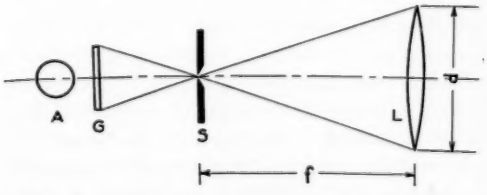

Fig. 6. Diagrammatic representation of the broad source mode. Source A and diffusion screen G are placed before the slit of a spectrograph. The screen may be opal or ground glass, or ground fused silica. Sometimes it is necessary to employ several ground surfaces. An alternative arrangement of apparatus is shown in Fig. 7.

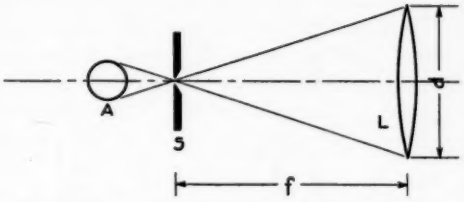

Fig. 7. Diagrammatic representation of the broad source mode. Source A is placed before the spectrograph slit. Since no diffusing screen is used, the source should be of uniform brightness.

such a distance, usually, that the horizontal angle subtended by it at the slit is equal to or greater than that subtended by the collimator lens. Such a large source angle insures maximum flux transmission, but, as in the case of the lens mode, it is usually better to sacrifice some intensity in order to realize the gain in resolution which comes from approaching coherent irradiation. (See Fig. 8.)

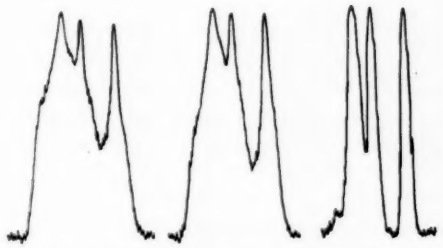

Fig. 8. Line shapes obtained with the broad source mode. The spectrograms whose photometric records are reproduced were made with a ground fused silica screen as secondary source stopped so that the collimator lens was (a) filled, (b) three-quarters filled, and (c) one-quarter filled, respectively. Hg λλ3650–63.

Any broad source may be considered to be composed of a large number of independent narrow sources, each of which produces a line of a certain shape and intensity. Through a study of lines produced by representative elements the advantage of a relatively narrow broad-source angle becomes apparent. Such a study has been made in the manner indicated by Fig. 9. Effectively, a broad source was screened so that only a small part of it was uncovered. A large number of spectrograms were made, each with a different element as the source. Lines produced with the elements indicated in Fig. 9 have been recorded microphotometrically and reproduced

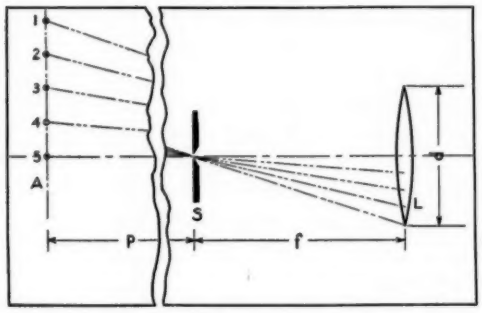

FIG. 9. Arrangement of apparatus employed in photographing the lines recorded in Fig. 10. Source A, approximately 1 mm in diameter was placed successively in positions 1–5 at a distance of 2 meters from the slit of the spectrograph. In position 1 a line drawn from the source through the slit would have met the collimator lens L at the rim near the prism base. Position 5 was on the optical axis of the collimator.

in Fig. 10. Similar results were obtained with elements on the other side of the axis. From these it is apparent that the central elements contribute the most to the central intensity; the lines are not only strong but they nearly coincide. The lines produced by the elements near the sides of the broad source are relatively weak and they are badly shifted in position. They therefore contribute to the width of the composite almost wholly. Intermediate elements contribute some to both width and central intensity, the distribution of the contribution depending on the relative distances of the elements from the axis. It is evident that an improvement in resolution must result from symmetrically decreasing the source angle, and that little loss in central intensity can occur until the angle has been decreased considerably.

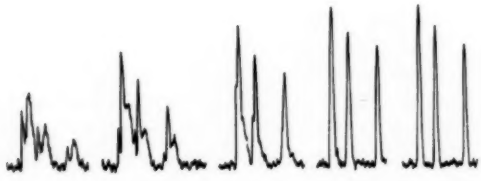

FIG. 10. Analysis of broad source mode. The above microphotograms of Hg λλ3650–63 were made from spectrograms obtained as indicated in Fig. 9. Exposure time, slit width, and mode were kept constant. The slit width was equal to the value which was critical for the coherent mode and correct source alignment. These examples serve also to illustrate the importance of source alignment in general. Note the extreme doubling and low maximum intensity resulting from placing the source far off the collimator axis.

Entirely similar studies made in the vertical source angle showed that the height of the source had comparatively little effect on the resolution.

It may be mentioned at this point that somewhat analogous analytical experiments have been performed in connection with the lens mode. In these the lens was masked by an opaque screen with a small hole which could be placed over any desired part of the lens surface. The results were essentially the same as those just presented.

IDEAL CONDITIONS FOR LINE INTENSITY MEASUREMENT

The problem of measuring line intensities differs from that of determining wave-lengths in that positions of peaks are of relatively small importance. Here the chief requirements are that there shall be sufficient resolution to insure independence of lines, i.e., no overlapping of primary and secondary maxima, and that the relationship between maximum intensity and total line flux shall be constant. The former condition can usually be satisfied by making the source angle sufficiently small, but the latter calls for line shape constancy. Any device or operation which in effect changes the mode is to be avoided. For example, if the lens mode is employed it is not permissible to vary the intensity by changing the lens aperture. Likewise, since slit width affects the line shape, this must be kept constant regardless of the mode.

While it is true that the photographic emulsion as a detector is accurate to within a few percent when used correctly, the measurement of its response often involves very large errors, sometimes of the order of one hundred percent. A part of this error can be traced to failure to satisfy the two chief requirements outlined above, but often a still greater part results from the actual measurement of density. If, for example, some radiation leaks past the edges of a line when the density is being determined, a value which is too low is obtained. Except with photometers of high resolving power, it is practically impossible to measure the maximum density of a sharp peak or of a very narrow line. This being true, it is quite common practice to photograph the lines with a slit width as great as the line separation will permit. The density of some portion of

the line, usually near the center, is taken as a measure of the intensity on the assumption that the line has a flat top, i.e., has uniform intensity distribution across the broadened peak. Actually, flat top lines are probably seldom obtained, the departure from that condition depending on a number of factors only one of which will be discussed here.

Assuming the broadened line to be symmetrical, there still remains the possibility of a more or less distinct diffraction structure. In general those modes which, with the critical slit width, yield the best line shapes are the least suitable for intensity measurements when wide slits are used. In the limiting case of the coherent mode, lines made with wide slits are composed of a number of fairly well resolved narrow components between which radiation could easily leak during the densitometry. It can therefore be concluded that flat top lines are most nearly obtained when the departure from noncoherent irradiation is not very great.

To illustrate how the line shape varies with the relative aperture of the condenser lens when a wide slit is used, a number of spectrograms were made under conditions which were identical except the size of stop at the condenser lens and the exposure time. The latter was adjusted to give practically equal densities, while the former was varied from 1.25 down to 0.01 times the relative aperture of the collimator lens. The arrangement of the apparatus and the results are shown in Figs. 11 and 12, respectively.

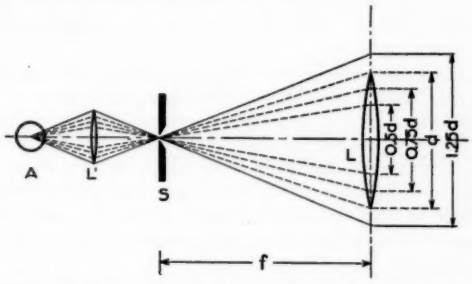

FIG. 11. Arrangement of apparatus employed in photographing the lines recorded in Fig. 12. An image of a part of source A was focussed by a variable stop lens L' whose maximum relative aperture was 1.25 times that of the collimator lens. A wide slit was used.

Although somewhat better line shapes might have been obtained with a still wider slit, the fact is not illustrated because very wide slits often cause the lines to overlap and are therefore of limited use.

We are greatly indebted to Professor A. C. Hardy and Mr. O. W. Pineo for their many helpful suggestions.

FIG. 12. Illustrating the departure from flat top line shape in the case of a wide slit. All spectrograms were made with apparatus arranged as shown in Fig. 11. From left to right the condenser lens aperture, referred to the collimator, was 1.3, 1.0, 0.75, 0.50, 0.25, 0.13, 0.06, 0.03, 0.015, and 0.01. In this example the closest approach to flat top shape was realized when the condenser lens was stopped to one-half the relative aperture of the collimator lens. Hg λ4047. The multiplicity obtained, when the lens aperture was greatly reduced, shows clearly that the slit width was nearly four times the critical value for the coherent mode.

Experimental Evidence for the Absence of Scattering of Light by Light*

F. L. MOHLER, *Bureau of Standards, Washington, D. C.*

(Received October 2, 1933)

A possible interpretation of certain theoretical relations is that photons will repel and scatter each other at a distance comparable with the wave-length of the radiation. To test this possibility, light from two projection lamps was brought to a common focus by two large aperture mirrors. The focal point at the center of an evacuated bulb was viewed at right angles to the beams of light by means of a viewing tube sighting into a black inclosure. There was no perceptible scattering of the light. The least perceptible light was measured and found to be about 5×10^{-9} of the brightness of the focal spot on a white surface. The radiation density at the focus was comparable to that in an inclosure at 2000°K through a volume of a cubic centimeter. From this it is estimated that if there is any interaction the scattering radius for photons of visible light is less than 5×10^{-9} cm or 10^{-4} of a wave-length.

IT is a generally accepted postulate of the wave theory of light that the waves are superposable; that is, that beams of light can cross without interacting in any way. However, it has, from time to time, been suggested that on the corpuscular viewpoint, there is a size or volume associated with a photon of a magnitude comparable to the cube of the wave-length. This magnitude is shown most simply by the uncertainty principle in the form

$$\Delta p\ \Delta q = h/2\pi,$$

where Δp is the uncertainty in the component of the momentum along a given direction and Δq is the uncertainty in position in the same direction. The maximum uncertainty Δp for light of a given frequency, ν, but random direction is $2h\nu/c$; hence the minimum uncertainty in position, Δq, is $\lambda/4\pi$. A particle of this diameter would have an uncertainty in position consistent with the uncertainty principle though it does not follow that photons are particles of this size. However, Meksyn[1] in a recent paper interprets certain wave-mechanical equations to mean that there is a volume $3\lambda^3/8\pi$, associated with a photon from which other photons are excluded. This would mean that two photons at a distance of about $\lambda/3$ must repel and scatter each other and that when two light beams cross there is some scattering of light in random directions.

Is this interpretation of the wave-mechanics equations consistent with experiment? It is a matter of common observation that scattering of light by light must be very slight. Otherwise when a bright light is brought to a focus the focal spot would be visible in empty space much as it is in smoky or dusty air. I have heard indirectly reports of negative experiments on simple scattering of light by light without change in wave-length but can find no published results which can serve as a quantitative test of the above views. For this reason a brief report of a negative result I have obtained may be of interest.

Two 400-watt Mazda projection lamps were brought to a common focus by two Mangin mirrors, the two beams meeting at 180°. The angular aperture of each mirror at the image was about one. The common focus was at the center of a glass bulb which was evacuated to remove all scattering by air and dust. The bulb was near the middle of a glass tube about a meter long and blackened metal tubes with suitable diaphragms inside and outside the glass tube formed a viewing tube and black inclosure. The line of sight along the axis of the tube was at right angles to the beams of light and in the plane of the images of the planar filaments. Each image showing the separate helices of the lamp filament was about equivalent to a uniformly illuminated area one centimeter across.

* Publication approved by the Director of the Bureau of Standards of the U. S. Department of Commerce.

[1] Meksyn, *Uncertainty Relations and Volume of Photons*, Phil. Mag. **15**, 592 (1933).

The observations were made from a darkened room and at the far end of the black inclosure was a small hole which could be illuminated by a flash lamp bulb so as to be barely visible. This served to fix the attention on the center of the field.

Under maximum illumination the limiting diaphragm in the viewing tube was barely visible against the black-body aperture and tests showed that this faint light came from the unfocussed light of the lamps. The middle of the field appeared completely dark and repeated tests showed that cutting off the focussed light from one lamp made no perceptible change.

A numerical estimate of the least perceptible light was made as follows: A black glass of transmission 3×10^{-3} was placed over the small hole at the far end of the black-body and the current in the lamp back of the hole was found which was at the limit of visibility under the actual conditions of the light scattering experiment. With the black glass removed the brightness of the small hole was measurable with an optical pyrometer and the limit of visibility was taken as 3×10^{-3} times this measured brightness.

The images of the separate helices of the Mazda lamps on a diffuse white surface had a brightness corresponding to a black-body at about 2000°K. This is about 2×10^{8} times the least perceptible brightness.

I will, rather arbitrarily, consider the photons effective in scattering visible light to lie in a range of 2000A around 5800A. The radiation flux at 2000°K in this range in an angle 2π is 1.2×10^{18} quanta per second. The flux of incident radiation is taken to be equal to this and the density of photons at the focus is then 4×10^{7} per cubic centimeter for each beam. If σ is the effective scattering area of a photon then a fraction $4\times10^{7}\sigma$ of the light from each beam will be scattered by the other in a cubic centimeter or a fraction $8\times10^{7}\sigma$ will be scattered from both. As less than 5×10^{-9} is scattered in about unit volume

$$8\times10^{7}\sigma<5\times10^{-9},\ \sigma<6\times10^{-17}\ \text{cm}^2$$

or the scattering radius is less than 5×10^{-9} cm.

This result definitely excludes the possibility that photons can scatter each other at a distance of $\lambda/3$ or 2×10^{-5} cm. The conditions of the experiment are, however, far from ideal and a test at least 1000 times as sensitive could be made with sunlight under good conditions. Hughes and Jauncey[2] have published the negative results of a careful search for the creation of a high energy photon from the collision of two low energy photons using sunlight and suitable color filters. Their results showed that the effective area for such a collision was less than 3×10^{-20} cm^2. A comparable sensitivity would be attainable for simple scattering, though the complete elimination of false scattering in the absence of filters would require elaborate precautions.

The effect looked for by Hughes and Jauncey can also be ruled out on the evidence that the converse process, the spontaneous creation of two photons from one is not observed. The radiation from distant stars maintains its spectral characteristics (apart from red shift) for periods of more than 10^{8} years.

[2] Hughes and Jauncey, Phys. Rev. **36**, 773 (1930).

Interference of Light in Metallic Films

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(Received July 14, 1933)

IN attempting to produce a film of manganese on glass by cathodic sputtering from a small sample of manganese, a series of brilliant interference fringes were observed in the film, when viewed by reflected light. Some of the preliminary observations have been previously reported.[1] A fuller description of these fringes will be given in this report, together with observations made on platinum films produced under like conditions. Recently similar observations have been reported by Overbeck[2] on sputtered films of tin, and by Wood[3] on films of sodium and potassium formed by condensation at liquid air temperatures.

When the sputtering is performed upon surfaces which are considerably smaller than that of the cathode, the deposit is fairly uniform over the receiving surface so that these fringes may escape notice. It is only when the cathode surface is much smaller than that of the receiving surface, that the narrow fringes are visible. Under these conditions the film which is produced, is one of variable thickness, and hence the conditions for the interference of light are similar to those obtained in the production of the well-known Newton's fringes. The shapes of these fringes depend upon that of the cathode, being circular for a round cathode as shown in Fig. 1 and Fig. 2(a), and approximately straight for a rectangular cathode as illustrated by Fig. 2(b).

The sputtering was carried out in a cylindrical chamber 16 cm in diameter and 28 cm long. The residual gas was hydrogen at a pressure ranging from two-tenths to four-tenths of a millimeter of mercury, the pressure being measured by means of a McLeod gauge. The hydrogen was produced electrolytically, washed in a caustic potash solution, then in distilled water, and finally dried over phosphorus pentoxide before being admitted to the discharge chamber. The cathode was encased in mica having either a circular or rectangular aperture facing the receiving surface. The circular aperture in the mica screen was 1.5 cm in diameter, while the rectangular aperture, used with the platinum cathode only, was 8 mm in width and 7 cm in length.

The receiving surface was a clear Pyrex glass plate, suitably placed upon a glass stool, the plate being about 1.3 cm from the cathode surface. Before sputtering the glass plate was carefully cleaned in concentrated caustic potash

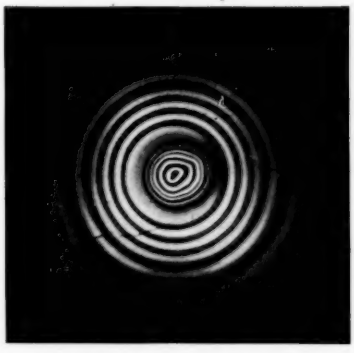
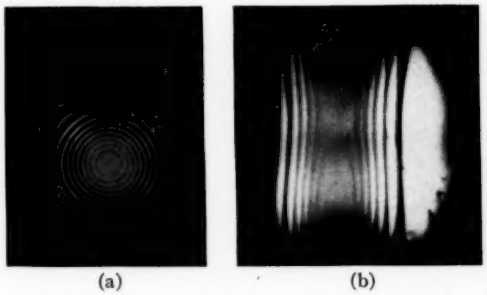


FIG. 1. Interference fringes in manganese film No. 4. $\lambda = 5461$A.



(a) (b)

FIG. 2. Interference fringes in platinum films. $\lambda = 5461$A. (a) Circular film. (b) Rectangular film.

[1] J. B. Nathanson, Phys. Rev. **41**, 373 (1932).
[2] C. J. Overbeck, J. O. S. A. **23**, 109 (1933).
[3] R.W.Wood, Bulletin Am. Phys. Soc. **8**, No. 2, p. 18 (1933).

solution, then washed in nitric acid, and finally rinsed thoroughly in distilled water, after which the plate was dried in an oven at 150°C. The mass of the sputtered film was determined by weighing on a highly sensitive balance. During the sputtering the edge of the Crookes dark space was kept out of contact with the glass plate, the discharge voltage varying from 2300 to 2500 volts. The time of sputtering varied with the metal, being nine to ten hours for manganese, and about two and one-half hours for platinum. The discharge current varied from 8 to 10 milliamperes for manganese, and 5 milliamperes for platinum.

The order of colors appearing at any point on the glass plate, as viewed by reflected light, seemed to be the same for manganese as for platinum, and was as follows: light brown, reddish brown, purple, blue, green, yellow, and back to light brown. This order is about the same as that reported by Edwards[4] on films of copper deposited upon aluminum and nickel, and by Rouard[5] on films of silver and gold upon glass. This sequence of colors repeats itself but with diminishing intensity. With continued sputtering the colored fringes move steadily outward from the center.

It seems quite plausible to suppose that the colored fringes are due to the interference of light between the front and rear surfaces of the films. If one assumes that these films possess the same optical constants and density as the metal in the bulk, then it is possible from the location of the fringes to estimate the mass of the film, and then to check the computations by weighing the film. Assuming that the phase differences in the light wave, which are introduced as the result of reflection and refraction at the front surface, and reflection at the rear surface of the film are the same at all points, then adjacent fringes of identical color must be located at points in the film differing in thickness by one-half of a wave-length as measured in the metal. Since the film thickness is a variable quantity diminishing to zero, we have then a step-by-step method for evaluating the thickness of the film, providing the thickness at some one point be known.

For example let us consider the thickness of the film at the outermost black fringe as viewed by reflected monochromatic light. To arrive at this value one must compute the phase differences by reflection and refraction at the air-metal and glass-metal surfaces. Drude[6] and Edwards[4] have shown how one might compute these values from Maxwell's equations, by applying the customary boundary conditions to the interface between two metals. The results might be applied to the interface between the glass and metal by assuming a value of zero for the coefficient of absorption of light in the glass. The phase change Δ_1 introduced by reflection at the air-metal boundary is given by $\tan \Delta_1 = 2nk/(n^2(k^2+1)-1)$, where n is the refractive index of the metal, and k is the coefficient of absorption per wave-length in the metal. Similarly upon refraction from air to metal, the phase change is given by $\tan \Delta_2 = nk/(n^2(k^2+1)+n)$. The phase change upon reflection from glass to metal to glass is given by $\tan \Delta_3 = 2n_1nk/(n^2(k^2+1)-n_1^2)$ where n_1 is the refractive index of the Pyrex glass. Upon refraction from metal to air the phase change is given by $\tan \Delta_4 = -nk/(n+1)$. For the outermost dark fringe the total phase difference between the ray reflected at the air-metal surface and that which has traversed the metal film, is 180°. Hence $\Delta_2+\Delta_3+\Delta_4+\Delta_5-\Delta_1=180$, where Δ_5 is the phase change due to the path traversed by the ray in passing back and forth in the film. The thickness t of the metal film at the outermost dark fringe is thus given by $t=\lambda'\Delta_5/720$. The thickness of the film at the next dark fringe would then be $t+\lambda'/2$, etc., where λ' is the wave-length in the metal. The value of t so computed was found to be from about 1/20th to 1/25 of the wave-length in the metal, the refractive index of the Pyrex glass being taken as 1.47.

In order to determine the positions of the fringes in the films by reflected light, the Pyrex glass plates were mounted vertically, and the positions of the dark fringes measured by means of a short focus cathetometer reading to 0.002 cm. The plates were illuminated by monochromatic light at nearly normal incidence. For circular fringes measurements were made along

[4] H. W. Edwards, Phys. Rev. **38**, 166 (1931).
[5] P. Rouard, C. R. **195**, 869 (1932).
[6] P. Drude, Ann. d. Physik **51**, 77 (1894).

two diameters at right angles to each other. In order to eliminate troublesome reflections from the rear of the glass plate, a glass prism was attached to the rear of the glass plate by means of cedar oil.

MANGANESE

When sputtering manganese, the glass plate directly underneath the center of the circular opening in the mica, was found to be free of any deposit. This was doubtless due to the intense heating of the glass plate produced by the cathode rays, the gas pressure being about 0.2 mm. The film was thus of variable thickness, being a maximum close under the edge of the circular opening, and diminishing to zero both towards the center and the edges of the glass plate. Hence a cross section of the film taken along any diameter, would be like a vertical cross section of a volcanic mountain, the center of the film corresponding to the crater of the volcano. Similar effects were observed with nickel which also sputters very slowly. A photograph of the manganese film taken with light of wave-length 5461A is shown in Fig. 1. Because of slight irregularities of the surface of the "crater" the inner fringes are not as regular as those corresponding to the more gentle outer slope.

A graph of the manganese film No. 4 is shown in Fig. 3. Sodium yellow light and mercury green light were used to locate the dark fringes. The thickness of the film plotted as ordinates is given in terms of $\frac{1}{2}$ wave-lengths in the metal. The value in cm can be obtained by multiplying the ordinate by $\lambda/2n$, where λ is the wave-length in air. The volume of the film was evaluated graphically from a large scale drawing by considering the crater-like film as built up of a large number of concentric cylinders, each of narrow wall thickness. The values of n and k used in the computations are 2.12 and 1.67, respectively, for a wave-length of 5461A, and 2.26 and 1.64, respectively, for a wave-length of 5893A. These are the values previously[7] obtained for the sample forming the cathode. The manganese sample was very pure, having been obtained from the commercial manganese by distillation in vacuum.[8] The thickness of the film at the outermost dark fringe was computed to be 0.038 λ/n and 0.039 λ/n for wave-lengths 5461A and 5893A, respectively.

The shift in position of the interference fringes with the wave-length of the light is shown in Fig. 4. In taking this photograph, each half of the film was alternately exposed to light of the mercury arc of wave-lengths 4359A and 5461A. A small black screen was used to cover half of the film while a photograph of the other half was being taken. It will be noticed that the diameters of the fringes for the shorter wave-length are somewhat larger than the corresponding ones for the longer wave-length, in accordance with the graphs of Fig. 3.

The results of the computations for manganese are given in Table I. The density of manganese was taken as 7.44 g per cm^3, this being the value

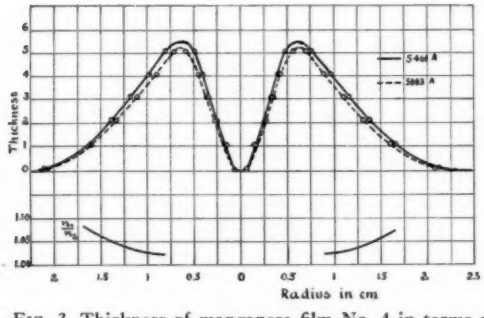


FIG. 3. Thickness of manganese film No. 4 in terms of the number of interference fringes. Below, the ratio of the refractive indices of the film for wave-lengths 5461 and 5893A.

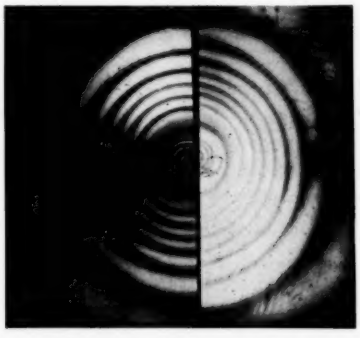


FIG. 4. Interference fringes in manganese film. Left and right halves of the film illuminated by light of wave-lengths 5461 and 4359A, respectively.

[7] J. B. Nathanson, J. O. S. A. **20**, 593 (1930).

[8] J. B. Friauf, Trans. Am. Soc. Steel Treating **18**, 213 (1930).

given by Preston[9] for alpha-manganese, the stable form at ordinary temperatures.

TABLE I. *Manganese films sputtered in hydrogen.*

Film No.	Diameter of film in cm	Maximum thickness $\times 10^8$ cm		Computed mass in mg		Weighed mass in mg
		5461A	5893A	5461A	5893A	
2	5.0	8.25	7.65	3.74	3.40	3.97
4	4.6	7.06	6.84	2.97	2.70	2.67

Inspection of Table I shows that the computed masses of the manganese films are in fair agreement with those obtained by weighing. It is noteworthy, however, that the computed masses with sodium yellow light are somewhat less than those computed with the aid of mercury green light. Similar results will be noted with platinum films, and will be discussed later.

PLATINUM

Because of the much higher rate of sputtering exhibited by platinum, the discharge current was kept less and the pressure of the residual hydrogen somewhat higher than for manganese. It was thus hoped to prevent the deposit from forming too rapidly. As a result the crater-like characteristics were absent in the platinum films. A photograph of one of these films is shown in Fig. 2(a), as viewed by light of wave-length 5461A. Ten dark fringes are visible, while only eight were observed when viewed with light of the lithium red flame.

Not all attempts to obtain platinum films showing the narrow interference fringes, resulted successfully, even though experimental conditions were kept the same. If the film produced exhibited the narrow fringes, then the film appeared clear orange red in color by transmitted light, otherwise the color exhibited was muddy brown. The platinum films adhered very firmly to the glass, and could be removed only by prolonged immersion in *aqua regia*. It was found that the thinnest part of the film adhered most closely to the glass, requiring many hours of immersion in the *aqua regia* to remove all trace.

[9] G. D. Preston, Phil. Mag. **5**, 1198 (1928).

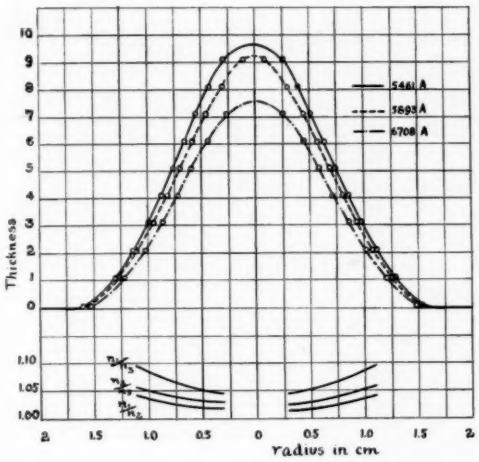


FIG. 5. Thickness of the circular platinum film in terms of the number of interference fringes. Below, the ratios of the refractive indices for the wave-lengths 5461, 5893 and 6708A.

The variation of thickness of one of the circular films is shown in Fig. 5, as evaluated by means of light of wave-lengths 5461A, 5893A and 6708A. The thickness in cm can be obtained by multiplying the ordinates by $\lambda/2n$. The optical constants of platinum were obtained from a graphical analysis of the values listed in volume 5 of the *International Critical Tables*. The refractive indices adopted are 1.85, 2.01, and 2.34 for light of wave-lengths 5461, 5893 and 6708A, respectively, while the corresponding values of the coefficients of absorption are 2.11, 2.10 and 2.08. The thicknesses of the films at the outermost dark fringe were thus computed to be $0.045\ \lambda_1/n_1$, $0.048\ \lambda_2/n_2$ and $0.053\ \lambda_3/n_3$, where the subscripts 1, 2 and 3 refer to wave-lengths 4561, 5893 and 6708A, respectively.

The results of the observations made upon two platinum films are summarized in Table II. The density of the platinum was taken as 21.45 g/cm^3.

The volume of the circular film was computed graphically by assuming the film as consisting of a large number of flat circular disks piled on top of each other. In the case of the platinum film obtained with the aperture in front of the cathode in the form of a narrow rectangle, the film was regarded as built up of a large number of rectangular disks. The length of the deposit on

TABLE II. *Platinum films sputtered in hydrogen.*

Film No.	Shape	Size	Maximum thickness $\times 10^5$ cm			Computed mass in mg			Weighed mass in mg
			5461A	5893A	6708A	5461A	5893A	6708A	
4	Circular	4 cm diam.	14.2	13.5	10.8	7.15	6.35	5.37	2.55
6	Rectangular	3.5×5 cm	8.9	8.2		12.63	11.46		4.09

the glass plate parallel to the long side of the rectangular aperture, was limited to 5 cm by means of strips of mica placed upon the plate.

The shift in the position of the interference fringes with the wave-length of the light is shown in Fig. 6, one-half of the film being illuminated by light of wave-length 4359A, and

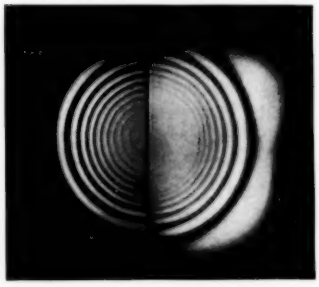


FIG. 6. Interference fringes in platinum film. Left and right halves of the film illuminated by light of wave-lengths 5461 and 4359A, respectively.

the other half by light of wave-length 5461A. As with manganese, the fringes of shorter wave-length are larger in diameter than the corresponding ones for the longer wave-length. The photograph was taken of the platinum film No. 4.

DISCUSSION

It is evident from an inspection of the results for the manganese and platinum films, that the difference between the computed and weighed masses of the films is much greater for platinum than for the manganese. Furthermore the computed masses seem to depend upon the wave-length of light, decreasing with increase in wave-length. This lack of agreement, especially for platinum, might be ascribed to several causes. In the first place there is some doubt as to whether the optical constants of the metal in the form of a thin film are the same as those of the metal in the bulk. Secondly, there is some question as to the physical state of the film.

A number of investigations of the optical constants of metals in the form of very thin films, has shown that these constants are the same for films as for the metal in the bulk, down to a certain thickness, below which the refractive index increases abnormally, while the coefficient of absorption decreases. Pogany[10] found this critical thickness to be 26 mμ for films of sputtered platinum. For silver and copper, this value was found by Richter[11] to be 15 mμ, while recently Schulze[12] found for gold films that the optical constants were the same as for the metal in the bulk down to a thickness of 2 mμ. The increase in the refractive index of a metal for exceedingly thin films is in the right direction for reducing the value of the computed mass of the films. However, the thickness of the films at the outermost dark fringe is only of the order of 15 mμ, so that the correction would be applicable to only a very small part of the whole film. This would account for only a small fraction of the difference between the weighed and computed masses.

There remains the question of the physical state of the films. It may be that under the conditions of the electrical discharge during sputtering, manganese and particularly platinum absorb hydrogen. This might not only decrease the density of the metal film, but might also modify the character of the dispersion of light within it. The fact that the computed mass seems to depend upon the wave-length of the light would indicate that the refractive indices of the films did not vary with the wave-length in the manner assumed for metals in the bulk.

The character of the dispersion can be shown

[10] B. Pogany, Ann. d. Physik **49**, 531 (1916).
[11] J. Richter, Ann. d. Physik **77**, 81 (1925).
[12] R. Schulze, Phys. Zeits. **34**, 24 (1933).

with the aid of the curves of Figs. 3 and 5. The computed thickness of the film at any point should be independent of the wave-length used in the measurement. Suppose y is the thickness at any point in terms of the number of half wave-lengths in the film. This number is given by the ordinates of the curves in the upper parts of Figs. 3 and 5. Hence for two wave-lengths λ_1 and λ_2 measured in air, we have that $y_1\lambda_1/2n_1 = y_2\lambda_2/2n_2$, where y_1 and y_2 are the ordinates of the curves for a given radius of the film. Hence $n_1/n_2 = y_1\lambda_1/y_2\lambda_2$. The terms of the right side of this equation being known, the ratio of the refractive indices can be computed. The results of these computations are shown graphically by the lower curves of Figs. 3 and 5. The refractive indices corresponding to wave-lengths 5461, 5893 and 6708A are given by n_1, n_2 and n_3, respectively.

In connection with these curves two things might be pointed out. First, the ratios are not constant but seem to approach constant values with increasing thickness of the film, the minimum values for n_1/n_2 being about the same for manganese as for platinum. Secondly, the ratios vary in such a manner with the wave-length as to indicate that the refractive index of the film for the violet end of the spectrum is greater than for the red end, as for the normal dispersion of a transparent dielectric of a high refractive index. This conclusion, if true, is contrary to the results of measurements made upon metals in the bulk with polarized light, as in the customary measurements of the optical constants of a metal. By means of the latter measurements, the refractive indices of manganese and of platinum are found to increase with increasing wave-length. This does not mean that the films are not metallic in character for they do possess some metallic luster.

With regards to the density of thin films, Murmann[13] found for silver films formed by evaporation and subsequent condensation in vacuum, that the density of the film of approximate thickness 15 mμ does not differ from that of the metal in the bulk. As reported recently in *Science*, an investigation by Finch and Quarrell[14] on the sputtering of thin films of aluminum upon platinum, showed that the aluminum assumed the crystalline pattern of the denser platinum, with the result that the actual density of the aluminum was greater than the normal one. Perhaps the reverse might be true of platinum deposited upon aluminum, with a decrease in the density of platinum, but this would hardly be the case of platinum deposited upon glass, since the latter does not possess the regular crystalline pattern of a metal.

In order to observe the effect of temperature on the films, the end of an electrically heated brass rod was pressed against the rear of the glass plates having the films. At a temperature of about 250°C, the straw color of the platinum film changed into the more characteristic gray color of the metal, with the disappearance of the fringes. This would tend to show that the platinum films which do exhibit the interference fringes might be in a colloidal form with hydrogen occluded. No change in color was noted for manganese. Finally a bare flame was applied directly to the manganese film. No perceptible change in the color of the film nor in the position of the interference fringes could be observed. One might conclude from this that the physical properties and hence the density of the manganese film remained unchanged, but that the density of the platinum in the film is considerably less than in the bulk. This would account for the fair agreement of the computed with the weighed mass of the manganese, and the poor agreement in the case of platinum.

It is hoped to extend the study of these anomalous films to other metals, so that greater light might be thrown on their nature.

[13] H. Murmann, Zeits. f. Physik **80**, 161 (1933).

[14] G. I. Finch and A. G. Quarrell, Science **77**, 11 (1933).

Neutral Value Scales. I. Munsell Neutral Value[1] Scale

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I. INTRODUCTION

THE Munsell *Book of Color*[3] and the *Manual of Color*[4] give a table of the values[1] and reflection factors which were used in preparing the 400 standard colored samples in the *Book of Color*, published as a revision of the *Atlas of the Munsell Color System*,[5] which embraced the pioneer work of A. H. Munsell, begun between 1900 and 1905.

In order to follow the steps which led to the adoption of the Munsell Value Scale we shall briefly trace the logic and the history of the experiments on which it was based, beginning with the work of Bouguer over a century and a half ago. The two value scales in most general use before the present work was initiated were representatives of two different corresponding types of laws, which may be called, respectively, the "logarithmic" and the "exponential" types.[6] The logarithmic type results from the older method of experimentation, but involves certain assumptions of questionable validity; the exponential type involves experimental difficulties, but is very direct and free from assumptions. The present Munsell standard value scale is intermediate in character, and results from experiments by two methods wholly different (both logically and technically), but with results in excellent agreement.

II. HISTORICAL SURVEY

The work by the first type of method began with P. Bouguer in 1760. By a simple method[7] he

[1] The relation between *Value* and *Brilliance* is as follows: "*Brilliance* is that attribute of any color in respect of which it may be classed as equivalent to some member of a series of grays ranging between black and white." (Report of the Committee on Colorimetry of the Optical Society of America, J. O. S. A. and R. S. I. **6**, 534 (1922).) *Value* bears a relation to the color attribute brilliance similar to that which the Fahrenheit and Centigrade scales, as measured with the various (gas, mercury, etc.) types of thermometers, bear to temperature. Value is the practical approach to brilliance afforded by reflection standards, the special case of brilliance so represented having the important difference of corresponding to a constant level of adaptation of the eye. More precisely, *value is the brilliance of the colors of a series of standards, varying in reflectance from zero to unity, in such a way that all visual intervals are uniform when viewed on a middle gray background by a light-adapted eye.* The standard Munsell value unit is defined as one-tenth the difference in value between a surface of 0 percent reflectance and one of 100 percent reflectance when both are viewed on a background whose reflectance is about 18 percent and illuminated by 22.8 f.c. of daylight (5200°K).

[2] In the prosecution of the researches described in this paper we wish to acknowledge the help we have received from various sources, particularly from Mr. Prentice Reeves, who planned a large part of the "J.N.D." work, and from Miss G. K. Walker, who was of great assistance in compiling the experimental data. Acknowledgment should also be made to Mr. I. G. Priest (now deceased) of the Bureau of Standards, whose keen interest in visual psychophysics and whose help and advice were an invaluable aid, and to Dr. D. B. Judd, of the same Bureau, who read the manuscript and gave very valuable criticisms.

[3] A. E. O. Munsell et al., Munsell *Book of Color*, on p. 46, Baltimore, 1929.

[4] F. G. Cooper, Munsell *Manual of Color*, on p. 33, Baltimore, 1929.

[5] A. H. Munsell, *Atlas of the Munsell Color System*, Boston, 1915.

[6] The Ostwald scale is of the logarithmic type corresponding to the Weber-Fechner law, considered below; the "Atlas" value scale of A. H. Munsell is of the exponential type presaged by the work of Plateau and others (see below). A third type of law, that of Adams and Cobb considered below, has not been applied to practical value scales.

[7] P. Bouguer, *Traité d'optique sur la gradation de la lumiere*, p. 51, Paris, (Lacaille) 1760. He used the following method. He set up a post between two candles and a white cardboard so that each candle cast a shadow on the latter. Shadow 1 was illuminated by candle 2, and shadow 2 by only candle 1, whereas the remaining area was illuminated by both candles. He varied the distances of the candles until one of the shadows disappeared. This occurred when the relative distances of the two candles was about 8.16 to 1, or such that the intensities of illuminations (as given by the "inverse square law") was about 1 to 66.67 (or 1.5 to 100). That is, the ratio of the brightness of the screen to that of the remaining shadow was 1.015.

determined the "just noticeable difference" (J.N.D.) between two brightnesses which under the conditions he used could be perceived. He found that the brightness B_2 which could be perceived as just greater than another, B_1, was 1.5 percent greater than B_1. That is, unless the ratio of the brightnesses was 1.015 or more the two brightnesses would sometimes appear the same. The fraction 0.015, which we now call the "Fechner fraction," was for Bouguer's own eye, and though it was different for his other observer's, the important fact was that it was on the whole *independent of the intensity of illumination;* that is, was more or less constant as the total brightness varied.

This *constancy* of the Fechner fraction was confirmed in the same year by Lambert, later by Steinheil and Arago, and after a century by Fechner,[8] who came to his conclusions both from visual experiments of Volkmann and from the work of Weber in other sense fields, these experiments using elaborations of Bouguer's simple method. The constancy of the Fechner fraction was also confirmed by Masson, using rotating disks, in 1845, and by Kraepelin forty years later, the method being logically the same as Bouguer's, but different in technique,[9] though these investigators obtained Fechner fractions of different magnitudes.

Fechner, assuming the constancy of the fraction now called by his name, and that this expressed a general psychophysical law (the "Weber law"), *and assuming further that a J.N.D. in sensation represents a unit sensation,* such units being capable of integration in the way *physical* units can be integrated, derived the well-known *logarithmic law* (now called the Weber-Fechner law),*

$$V = c \log R + k, \qquad (1)$$

where V is the sensation, R the physical stimulus (reflectance in the case of reflecting papers), and c and k are constants.

Criticisms of these laws soon followed. Insofar as we definitely know the facts, the range of illumination intensities covered by the experiments described had the extreme ratio of only about 100 to 1. Volkmann, in 1863–64, using Bouguer's general method, and Helmholtz using Masson's,[10] found that *the Fechner fraction* **decreased** *with increase of brightness;* and Aubert,[11] using both Bouguer's and Masson's methods, and a range of brightnesses in the ratio of 10,000 to 1, confirmed this conclusion for low brightnesses and found that a minimum value of the Fechner fraction of 0.0054 was obtained at brightnesses somewhat less than that of diffuse daylight.

A method of experimenting totally different logically, and leading to a result at variance with the Weber-Fechner law, was initiated by Plateau[12] in 1873. The experimental basis of Plateau's work may have been somewhat insecure, but the work is of interest because he introduced the method which we shall call the value-step method, and because he promulgated a new relation, an exponential (rather than a logarithmic) one, for the relation between value and reflection. He asked eight artists to paint a gray midway between black and white. Although there was no control of the illumination, the result was that the eight grays were practically of

[8] J. H. Lambert, *Photometria,* Augustae Videlic, 1760. C. A. Steinheil, *Elemente der Helligkeitsmessungen am Sternhimmel;* Abhandl. k. bayr. Akad. Wissensch., Math.-Phys. Cl. **2**, 1 (1837). Arago, *Oeuvres completes,* Paris, X; 1858. G. T. Fechner, *Über ein wichtiges psychophysisches Grundgesetz u. d. Beziehung z. Schatzung d. Sterngrossen,* Abhandl. k. sachs. ges. Wissensch., Math.-phys. Kl. **4**, 455 (1859). G. T. Fechner, *Elemente der Psychophysik,* Leipzig; 1860. The last-named obtained the Fechner fraction 0.010.

[9] M. A. Masson, *Études de photometrie electrique;* Ann. chim. et phys. [3] **14**, 129 (1845). E. Kraepelin, *Zur Frage der Gültigkeit des Weberschen Gesetzes bei Lichtempfindungen;* Phil. Stud. **2**, 306–26 (1885). E. Kraepelin, *Nachtrag zu der Arbeit u. d. Gültigkeit des Weberschen Gesetzes bei Lichtempfindungen;* Phil. Stud. **2**, 651–57 (1885). Masson introduced the method of using a series of white disks each with a small black sector. When the disk was rotated a faint gray ring was seen on the white background. The size of the black sector giving a just perceptible gray ring furnished a determination of the Fechner fraction. Later experimenters modified this technique in various ways, in part for the purpose of eliminating certain contrast effects which might modify the results.

* This law can have no meaning for $R=0$. See the discussion of the Delboeuf law below.

[10] A. W. Volkmann, Physiol. Untersuchungen im Gebiete d. Optik **1**, 56 (1863). H. Helmholtz, Handb. d. physiologischen Optik, Hamburg and Leipzig, 1886.

[11] H. Aubert, *Physiologie der Netzhaut;* pp. 52–82 (1865).

[12] J. Plateau, *Über die Messung physischer Empfindungen u. d. Gesetz welches die Starke d. erregenden Ursache verknupft,* Pogg. Ann. **150**, 465–76 (1873).

the same reflectance. Assuming that the *ratios* of the sensations produced by pairs of reflecting surfaces are independent of the illumination, and further assuming that the observers pick a mid-gray such that $V_2 : V_m = V_m : V_1$ (where the subscript m refers to the mid-gray and 2 and 1 to the pairs of grays), Plateau derived the law

$$V = kR^c, \tag{2}$$

where k and c are constants. This we shall call the *Plateau law.*[13] In the same year Delboeuf,[13] using Plateau's general method and a technique somewhat similar to Masson's,[9] obtained results which could be expressed by a modification of the Weber-Fechner law. Delboeuf's modified law was

$$V = c \log [(R+k)/k], \tag{3}$$

where c and k are constants. Plateau retracted his own law in favor of Delboeuf's, and later Lehman and Neiglick[14] modified Plateau's method in such a way as to eliminate the effect of contrast and confirmed approximately Delboeuf's law.

On the other hand Breton,[15] using Plateau's value-step method, and a range of brightnesses from that of black velvet at ordinary illumination to that of a white surface at dazzling brilliance, found 5 steps of value which did not conform to the logarithmic laws, and could be expressed by an exponential law of the Plateau type, where the exponent c has the value 0.5.

Between 1888 and 1894, Merkel[16] performed experiments by *both* Bouguer's J.N.D. and Plateau's value-step method. By the former he found that the Fechner fraction decreased with increase of brightnesses in the ratio 200 : 1 but was constant for a change in the ratio 3000 : 200; and by the latter method he found that the mid-gray corresponds to a point *between* the geometric mean corresponding to a logarithmic law and the arithmetical mean corresponding to an exponential law. Ebbinghaus and Ament[17] obtained similar results, the former by using 50 gray papers to determine an 8-step value scale. Fröbes got conflicting results, but in one series of experiments confirmed Merkel's conclusion; and Stefanini[18] pointed out that Merkel's results agreed with Breton's form of the Plateau law.

The work of the later investigators was signalized both by a return to the J.N.D. method and by a more explicit statement of the conditions under which the experiments were conducted. A set of widely quoted experiments were those of König and Brodhun,[19] which extended over a range of brightnesses from the lowest visible under the conditions to a dazzling brilliance. Since these experiments were conducted with a dark field surrounding the test spots of light, and the observers were dark-adapted, the results are not directly comparable with those of the present investigation. Hence they will be reported only in a general way and in the form of Fig. 1, along with the results of the experimenter whose work is next considered.

Between 1913 and 1916 Cobb[20] reported the

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[13] By way of comparison with the Plateau law, it can be shown that the Weber-Fechner law results from assuming that observers pick a mid-gray in such a way that it gives equal *differences* (rather than equal ratios) of sensations when compared with black and white; i.e., so that $V_2 - V_m = V_m - V_1$. Delboeuf, Bull. de l'Acad. royale de Belgique **34**, 250, 261 (1872), pointed out that according to the Weber-Fechner law, a small positive value of R corresponds to a *negative* value of V, and zero value of R to an *infinitely negative* value of V, which is an absurdity. Further, his results were not entirely independent of the illumination, as required by the Weber-Fechner law.

[14] A. Lehman, *Über die Anwendung der Methode der mittleren Abstufung auf den lichtsinn;* Wundt's Phil. Stud. **3**, 497–533 (1886). H. Neiglick, *Zur Psychophysik des Lichtsinns;* Wundt's Phil. Stud. **4**, 28–111 (1888).

[15] P. Breton, Comptes Rendus **105**, 426–29 (1887).

[16] J. Merkel, *Die Abhangigkeit zwischen Reiz und Empfindund,* Wundt's Phil. Stud. **4**, 541 (1888); **5**, 499 (1889); **10**, 239 (1894).

[17] Ebbinghaus; *Grundzuge der Psychologie,* I, 497, Leipzig, 1902. W. Ament, *Über das Verhaltnis der ebenmerklichen zu den ubermerklichen Unterschieden bei Licht- und Schallintensitaten;* Wundt's Phil Stud. **16**, 135–96 (1900).

[18] J. Fröbes, *Ein Beitrag über die sogenannten Vergleichungen ubermerklicher Empfindungsunterschiede;* Zeits. f. Psychologie **36**, 304–80 (1904). A. Stefanini, *Sulla legge di oscillazione dei diapsson e sulla misura dell' intensita del suono;* Atti della R. Acad. Lucchese **25**, 305–400 (1889).

[19] A. König, and E. Brodhun, *Experimentelle Untersuchungen über die Psychophysiche Fundamentalform in Bezug auf den Gesichtssinn;* Gesammelte Abhandl. z. Physiologischen Optik, p. 135–39. J. Blanchard, *The Brightness Sensibility of the Retina,* Phys. Rev. **11**, 81–99 (1918); obtained results agreeing in general with König and Brodhun's (and Aubert's).

[20] P. W. Cobb, and L. R. Geissler, *The Effect on Foveal Vision of Bright Surroundings, I;* Psychol. Rev. **20**, 525–47 (1913). P. W. Cobb, *The Effect on Foveal Vision of Bright*

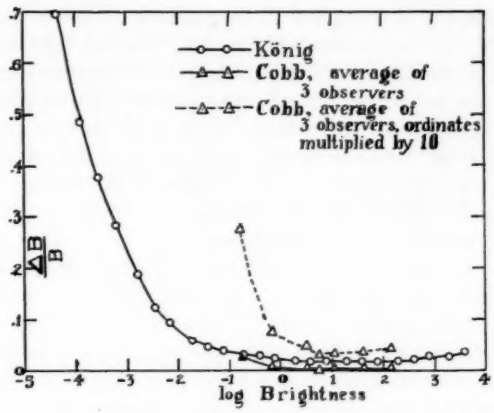

FIG. 1. Showing variation of the Fechner fraction with logarithm of brightness.

results of investigations made to determine the sensitivity of the eye at different brightnesses, under carefully controlled conditions of light adaptation and light surrounding field. Three observers viewed a slightly yellowish test field subtending a visual angle of 2.0° by 2.6°, surrounded by the illuminated interior of a polyhedral box, the brightness of which was 5.5 millilamberts. The test field brightnesses ranged from 0.172 to 139.0 millilamberts. The results are shown in Fig. 1.

We may summarize the results of these two modern investigations as follows: For both light and dark adaptation, as the brightness is increased the Fechner fraction decreases very rapidly at low brightnesses and increases slightly at high brightnesses. For a middle range (4 to 200 ml in König's work and a smaller range in Cobb's) the Fechner fraction in dark adaptation remains constant and in light adaptation goes through a minimum at a brightness equal to that of the surrounding field.

The bearing of these and the foregoing results on the laws from which value scales may be computed may be expressed as follows: Neither the Weber-Fechner logarithmic (or its modification, the Delboeuf law) nor the Plateau-Breton exponential law adequately represents the results for the experimental conditions of the investigations cited. The difference between these two may be most clearly shown by computing from them the reflectance of the mid-gray, taking the lightest practically obtainable white (a magnesia surface reflecting 98 percent) and the darkest practically obtainable black (velvet of reflectance 0.4 percent). Both laws give results far from 50 percent; the logarithmic law gives slightly over 6 percent, and the exponential law with Breton's value of the exponent (=0.5), the other constant assumed as unity, gives about 28 percent. We shall see that for the conditions of the present work, the mid-gray has a reflectance between the last two figures.

A third type of law was proposed by Adams and Cobb[21] to express the results of Cobb.[20] This was

$$V = cR/(R+R_a), \qquad (4)$$

where V is the sensation measured from a point of reference V_0, c is a constant, R is the test field brightness, and R_a is the adaptation-brightness. This is the only equation among those which have been proposed which *takes account of one of the chief variable* conditions affecting the relation between the values of a series of reflecting standards and their reflectances, namely of the adaptation-brightness. These conditions include: (1) The adaptation-brightness, which depends upon the brightness of the surrounding field and the state of adaptation of the observer's eye and its consequent size of pupil; (2) the range of illumination intensities for the samples and background; (3) the sizes of stimulus-fields used; (4) the color of the illuminant—often neutral or yellowish; (5) the different units used for the measurement of sensation; (6) the normality or departure from normal of the observers. Of these, the first one is found to be the most important.

Between 1900 and 1905, A. H. Munsell searched assiduously among psychologists, physiologists and physicists for an opinion as to what the true value scale should be. Failing at the time to find a reliable consensus of opinion, he was led as a temporary expedient to the adoption

Surroundings, II; Psychol. Rev. **21**, 23–32 (1914). P. W. Cobb, *The Effect on Foveal Vision of Bright Surroundings, III;* J. Exper. Psychol. **1**, 419–25 (1916); *IV,* J. Exper. Psychol. **1**, 540–66 (1916).

[21] E. Q. Adams, and P. W. Cobb, *The Effect on Foveal Vision of Light (and Dark) Surroundings, V;* J. Exper. Psychol. **5**, 39–45 (1922).

of a scale corresponding to the results of Plateau and Breton. He invented a portable photometer[22] with which he calibrated the samples making up the value scale of the *Atlas*.[5] Following his untimely death in 1918, steps have been taken to determine the neutral value scale corresponding to the practicable conditions for the use of a system of reflecting papers. The remainder of this paper covers the experimental work in this laboratory which began in 1923, and was composed of two parts, each following one of the two general methods already listed; and a discussion of the results.

III. Introduction to the Experimental Section

In the historical survey it was apparent that further research was necessary to determine the effect of various experimental conditions on the value-reflection relation, and to determine whether or not the J.N.D. was acceptable as a sensation unit. In the present investigation a study of this relation has been made under *light-adapted* conditions. The experimental conditions listed above were chosen with the purpose of applying the data to the specification of practical value scales to be used on standard color charts. In Section IV (which follows) the J.N.D. method was used, while in Section V the value-step method was applied. A comparison of the results of the two methods sheds (a favorable) light on the validity of the J.N.D. as a unit of sensation. The latter method was also used in two supplementary investigations, which will also be reported, in which studies were made of the effect on the determinations of changes in the intensity and quality of the illumination and in the reflectance of the background.

In the value-step method, entire value scales were set up, each being composed of nine reflecting gray samples. The samples were ones selected at will by each observer from a large number of samples exhibited on nine strips. By moving any one of the strips, samples very close together in value were made visible through an aperture. In this way 26 sets of observations of value scales as a whole were made by carefully trained and selected observers. The results (see Section V) gave a surprisingly close agreement with those of the J.N.D. method, a description of which follows.

IV. Just-Noticeable-Difference Experiments

Briefly, the J.N.D. method consisted in the comparison of two small contiguous gray areas, differing slightly from each other in brightness. Six experienced observers determined the brightnesses corresponding to just noticeable differences in value at five value levels (0.2/, 2.4/, 5.0/, 6.7/ and 9.0/). From these measurements, on certain assumptions to be stated, the neutral value scale already reported[3, 4] was computed.

1. Description of apparatus

The apparatus used in this investigation was designed to permit of a determination (a) of the initial threshold for neutral light (i.e., of the least amount of light that appears just noticeably different from the sensation produced by zero light stimulus), and (b) of the "differential threshold" or just noticeable difference for neutral light at any desired level of brightness. We shall first describe the apparatus used to determine the initial threshold and the J.N.D. at very low brightnesses.

The general arrangement of the apparatus is shown diagrammatically in Fig. 2. A circular aperture A cut in the screen S formed the boundary of the stimulus field. At the observer's

[22] This photometer operated on the "cat's-eye" principle, that is, the area of the variable aperture through which a reflecting standard was illuminated with decreasing intensity until the sample was matched (or the movement of the brightness indicator) was proportional to the square of the diagonal of the (square) aperture. Equal areas of aperture or equal movements of the indicator corresponded approximately to equal sensation (rather than equal reflection) differences. This relation corresponded further to the simple rule: The percentage reflectance of any one of the ten steps of value in the Munsell "Atlas Value Scale" is obtained by multiplying this value by itself. For example, value 7 gives 7×7 or 49 percent. This earlier Munsell value scale was carefully examined by the U. S. Bureau of Standards, with the results reported by Priest, Gibson and McNicholas (Bur. Stand. Tech. Pap. No. 167, p. 27; 1920) as follows:

Value level	1/	2/	3/	4/	5/	6/	7/	8/	9/	
Value×value	1	4	9	16	25	36	49	64	81	%
B. S. findings	2	4	9	16	23	34	46	60	72	%

Considering the drawbacks of the photometer resulting from its portable construction, the results are very satisfactory.

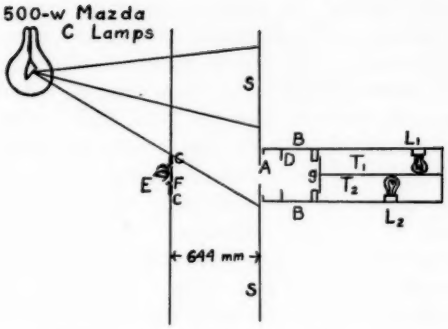

FIG. 2. Apparatus for J.N.D. experiments at low brightnesses.

eye E, 644 mm from the aperture, it subtended a visual angle of 4° 40′. The screen S which formed the surrounding field was painted a matt gray. Its reflectance was 19.1 percent. For the illumination of the observation room six 500-watt Mazda C lamps, operated on a 120-volt city circuit, were used. The position of these is shown in the figure. The color of the light was chosen to accord with that suggested tentatively by Priest[23] as the standard of neutral ("white") light. Priest says: "The most probable average value for the yellow-blue transition temperature . . . is about 5200°K. That is to say, *based on these limited data*, the best representative temperature for a Planckian radiator, in order that its radiant energy shall evoke the gray (neutral) sensation in the average individual, is 5200°K." To obtain the desired chromatic (color) quality a pair of "artificial daylight" spectacles of Macbeth Daylight glass especially chosen for this purpose were worn by the observer while observing. The color temperature of the room illumination may be taken as approximately 2848°K. The apparent color temperature for an observer wearing the glasses was determined at the Bureau of Standards. It was found to be 5270°K for the right glass and 5180°K for the left. The second figure, obtained by direct measurement with a rotary dispersion colorimetric photometer, was checked by computation from spectral transmissions of the color temperature of the source viewed through the left glass, which gave 5240°K. The transmission of the glasses when used with a source of 2848°K was found from the spectrophotometric data to be 18.5 percent. The illumination of the surrounding field from the six 500-watt lamps, when corrected for the absorption of the blue glasses, was approximately 22.8 f.c.

B in Fig. 2 was an enclosure lined throughout with black velvet except for a circular section on one side, which consisted of a piece of ground glass. The stimulus field was illuminated by transmitted light. The upper and lower halves of the ground glass were illuminated by means of the lamps L_1 and L_2 which travelled along the enclosed tracks, T_1 and T_2. The observer saw through the aperture A two semicircular fields whose brightnesses depended upon the distances of the lamps L_1 and L_2 from the ground glass screen. L_1 and L_2 were small automobile headlight lamps each operated on a storage battery in series with a rheostat and an ammeter. Before being used the lamps were carefully seasoned. In order to reduce the two halves of the stimulus field to sufficiently low brightnesses one or more of a series of filters of different transmissions were placed over the ground glass screen. Each of these consisted of several thicknesses of white typewriter paper in conjunction with a blue and a green gelatine filter chosen so that the stimulus field would appear neutral in color to the observer wearing the blue glasses.

2. Calibration of track and filters

The exact distances of the lamps from the ground glass were read on scales attached to the outside of the track housing by means of pointers attached to the lamps. A calibration curve of brightnesses of the stimulus field for various positions of the lamp was made by means of a Macbeth illuminometer. Each of the filters used was calibrated by making readings of the brightness of the ground glass with and without the filter. Special precautions were taken to prevent any light from the observation room from falling on the ground glass surface. The transverse septum D placed across the center of the box cut off the direct light from the lamps mounted in the ceiling of the observation room. A black cardboard screen about two feet square

[23] I. G. Priest, *The Spectral Distribution of Energy Required to Evoke the Gray Sensation*, Bur. Stand. Sci. Pap. No. 417; 1921. In the quotation, the word in brackets has been inserted by the authors.

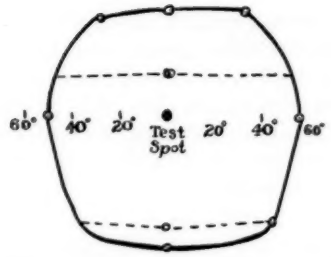

FIG. 3. Showing limitation of field of view by blue glasses. Broken curve with cardboard screen; unbroken curve, without cardboard screen.

hung directly in front of the observer cut off light reflected from the walls. The stimulus field was viewed through an oblong hole in this screen, which limited the observer's field of vision somewhat, as shown in Fig. 3, by means of the dotted line. The solid line shows the approximate extent of the field for an observer wearing the blue glasses.

For the determination of just noticeable differences at higher brightness levels a different arrangement of apparatus was used in which the two halves of the stimulus field were formed by reflecting surfaces illuminated by 22.8 f.c. of light from the six 500-watt lamps. This apparatus is shown in Fig. 4. The observer's position relative to the stimulus opening, as well as the surrounding field and its illumination were as described before. The lower semicircular half of the aperture *A* was filled by a fixed neutral gray paper *P* of the reflectance at which it was desired to determine the J.N.D. Provision was made for varying the apparent reflectance of the upper half of the field continuously and very gradually so that a determination could be made of the reflectances which appeared respectively just noticeably lighter and just noticeably darker than the lower half of the field. This variation was obtained by means of a pair of Maxwell disks. One of these disks consisted of a neutral gray paper of low reflectance, the other of a paper of a higher reflectance than the neutral sample *P*. The relative proportions of the exposed surfaces of these two disks were varied continuously while they were rotating; by this means a gradual increase or decrease of the reflectance of the upper half of the field was obtained. The mechanism for rotating the two Maxwell disks and for varying the relative proportions of their exposed areas was a modification of that of the direct reading rotating disk photometer which forms a part of the Keuffel and Esser color analyzer.[24] In the sector photometer described in the reference the cam slot extends through only 90°; in our instrument, however, it extends through 360°. In the form of the instrument used in this research, two disks of aluminum, on which neutral gray papers of the desired reflectance were mounted, were fastened to the instrument by means of small screws. When the knurled ring (33 of the figure of reference 24) was rotated by the operator the exposed area of one disk was increased while that of the other disk was decreased. The scale of the instrument gave directly the area of the front disk relative to the total area. The precision of the instrument was such that a change of one percent in disk area, corresponding to a negligible change in reflectance, could be measured accurately. In Fig. 5 the disks are shown in place behind the circular aperture *A*. The disks shown in this figure are in the position corresponding to a scale reading of 75 percent. In order that the reflectance of the comparison field might be varied by very gradual amounts above and below that of the standard field, separate pairs of disks were used with each of the neutral papers which composed the standard field. On these were mounted papers of reflectances slightly greater and slightly less respectively than that of the paper in the fixed

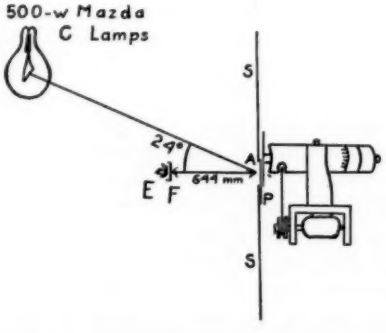

FIG. 4. Apparatus for J.N.D. experiments at high brightnesses.

[24] C. W. Keuffel, *A Direct Reading Spectrophotometer*, J. Opt. Soc. Am. **11**, 407 (1925); see the figure on p. 406.

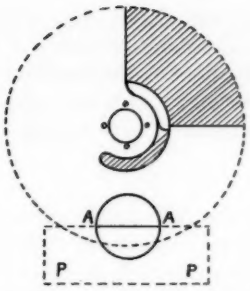

FIG. 5. Showing disks in position behind aperture.

standard field. The approximate reflectances of these papers are given in Table I. In column 4 of this table is given for each pair of disks the minimum change in reflectance that could be obtained and reproduced, i.e., the change in reflectance corresponding to one percent change in relative area.

TABLE I. *Showing the smallest change in reflectance obtainable with pairs of rotating disks.*

Reflectance of Standard field (percent)	Reflectance of Lighter disk (percent)	Reflectance of Darker disk (percent)	Smallest significant change in reflectance that can be obtained with each combination of disks (percent)
72.9	82.2	58.5	0.034
35.7	40.2	28.5	0.103
17.9	20.7	10.4	0.12
4.2	7.23	2.4	0.24

The reflectances of the neutral papers were determined by several different methods. The integral reflectances were measured by means of a Taylor reflectometer with a Flicker photometer attachment. Since the papers when in actual use were illuminated at an angle of approximately 24° and were viewed normally, and it was not known to what degree of accuracy they might be considered perfectly diffusing, their reflectances were also determined with a Macbeth illuminometer under the actual conditions of illumination used in making the observations. As was mentioned previously, the scale of the disk instrument gave the area of one of the disks relative to the total area. In order that these readings might be converted into reflectance, calibration curves and tables were obtained for each pair of disks as follows. With the disk machine in position behind the circular aperture, determinations of reflectances were made with the Macbeth illuminometer for settings of the instrument corresponding to the following areas of the lighter disk: 100, 90, 50, 10, 0 percent. These were plotted against scale readings, and the straight line so determined served as a calibration curve.

3. Procedure

The general method of experimentation was to determine at a number of levels of reflectance the increase and decrease in reflectance required to give a J.N.D. in value. In order to obtain specifications for a value scale which shall cover the range from the threshold to 100 percent reflectance, it is necessary to determine either directly or indirectly the number of J.N.D's from the threshold of value (sensation) to the value level corresponding to a reflectance of 100 percent. In this investigation no determinations were made for reflectances greater than 70 percent. It is possible to obtain data above this point by extrapolation between 70 percent and 100 percent as will be seen from the curves to be presented later. Extrapolation at the lower end of the curve is, however, attended with greater uncertainty. For this reason determinations of the J.N.D. were made for each observer at and just above the point corresponding to his initial threshold.

At the beginning of each day's observations the illumination falling on the stimulus opening and the surrounding field was measured with a Macbeth illuminometer to see that no great change in intensity had occurred. These measurements made throughout the entire period of investigation showed an average of 22.8 f.c. illumination. The mean variation was from 21.6 to 24.0 f.c., and the extreme range of variation from 18.5 to 26.8 f.c.

A period of 10 minutes was allowed for the observer to become adapted to the illumination conditions of the experiment before observations were made. Each observer made a series of practice readings before any final data were taken. A single set of observations consisted of 6 separate determinations of the reflectances which appeared respectively just noticeably lighter and just noticeably darker than the fixed lower field. From 4 to 7 sets of observations were made at

each level of reflectance and at the initial threshold.

The initial threshold and the J.N.D. just above it were determined as follows. The lamp on the upper track was turned out; the lamp on the lower track was then moved slowly toward the ground glass screen until the lower field appeared just noticeably lighter than the darkened upper field. Six such readings were made in order to determine the proper position of the lamp required to give the initial threshold. A determination of the J.N.D. at this point was then made as follows. Both lamps were turned on. The lamp on the lower track was placed at the distance corresponding to the initial threshold. The lamp on the upper track was then moved slowly toward the ground glass screen until the upper field appeared just noticeably lighter than the lower. It was then moved slowly away from the screen until the upper field appeared just noticeably *darker* than the lower field. As before each set of observations consisted of 6 such determinations.

In calculating the J.N.D. from the data obtained in a given set of observations, the average reflectances which appeared just noticeably darker and just noticeably lighter than the standard were first obtained. One-half the difference between these two quantities then gave the difference in reflectance corresponding to a J.N.D. at this level of reflectance. This magnitude will hereafter be designated as ΔR and the level of reflectance to which it corresponds will be designated as R. Similarly, when the results are expressed in brightness units (millilamberts) instead of in reflectance units, the symbol ΔB will be used to represent the difference in *brightness* corresponding to a J.N.D. in value at the brightness level B.

The determinations were made by 6 observers possessing good vision. A complete ophthalmological examination was made of the eyes of the observers (except observer W.L.V.) The observers had normal visual acuity (20/20) or better when wearing the glasses prescribed.[25] The results of the other tests (range of accommodation, pupillary reaction, size of pupil, extent of bulbar excursions, tension, transparency of media of eye, and the condition of the fundus, conjunctiva, iris, and tear ducts), with the exception of those for muscle inbalance, were normal in all cases.

[25] The strengths of the correcting spheres or cylinders worn by four of the five observers were equal to or less than one diopter. The glasses worn by C. E. B. were O. D. −2.5 sph., −3 cyl. axis 165, O. S. −4.25 sph., −2 cyl. axis 10.

4. Results

Table II gives ΔB at the various brightnesses for each of the 6 observers. These data were obtained by the method described in the preceding section. The results are shown graphically in Fig. 6, in which ΔB is plotted as ordinate and the corresponding B as abscissa. The absolute magnitude of the J.N.D. is seen to be very different for the different observers. The curves are, however, of the same general shape; in all of them we find that the slope first decreases and then increases with increase of B. Two of the six curves are approximately straight lines between about 4 and 18 ml; i.e., in this region ΔB is directly proportional to B, as required by the Weber law. In the case of the other four observers, however, the Weber law cannot be said to hold over any considerable range. A further demonstration that the data are not in accord with the Weber law is given in Fig. 7. In this figure $\Delta B/B$ is plotted against log B in millilamberts. The data from which this curve was plotted are given in Table III. According to the Weber law, $\Delta B/B$ should remain a constant as log B is varied. If this were the case the points plotted in Fig. 7 would lie on straight lines parallel to the abscissa. The curves show on the contrary that the ratio $\Delta B/B$ decreases at first rapidly and then more slowly with increase of brightness. Between B 7.8 ml and B 24.5 ml the decrease in $\Delta B/B$ is very slight.

We shall next describe the method used in calculating, from the experimental data in reflectance units given in Table II, the specifications for a value scale of 10 equidistant sensation steps. The general procedure used was to determine by graphical methods a series of reflectances between 0 and 100 percent, which were separated by equal numbers of just-noticeably-different steps. If $1/\Delta R$ is plotted as ordinate and the corresponding R as abscissa, and a smooth curve is drawn through the points, then the area under this curve between any two reflectances gives the number of just-noticeable-

TABLE II. *Just noticeable differences at five levels of reflectance.*

(Data given in both reflectance and brightness units. A reflectance of 100 percent corresponds to a brightness of 24.5 ml.)

L.L.S.		G.K.W.		A.E.O.M.	
R	ΔR	R	ΔR	R	ΔR
0.0209%*	0.009685%	0.0199%*	0.01097%	0.051%*	0.013%
3.13	0.08429	3.29	0.177	4.33	0.2258
18.3	0.25	18.6	0.52	16.90	0.47
35.9	0.2715	36.1	0.783	34.8	0.62
76.4	0.615	76.0	1.55	74.1	1.09
B	ΔB	B	ΔB	B	ΔB
0.005135	0.00238	0.004883	0.002692	0.01251	0.00319
0.7365	0.0214	0.8079	0.04343	1.063	0.0554
4.492	0.0614	4.563	0.1276	4.148	0.1153
8.820	0.06665	8.859	0.1922	8.54	0.1521
18.76	0.151	18.65	0.382	18.2	0.2675

W.L.V.		C.E.B.		P.R.	
R	ΔR	R	ΔR	R	ΔR
0.025%*	0.015%	0.0145%*	0.014%	0.021%*	0.0055%
4.22	0.1495	4.39	0.286	4.10	0.21
17.1	0.310	17.9	0.43	18.59	0.545
34.9	0.5285	35.5	0.734	36.26	0.735
73.7	1.1	71.9	1.39	68.35	1.122
B	ΔB	B	ΔB	B	ΔB
0.00614	0.003681	0.00356	0.00344	0.005155	0.00135
1.036	0.03669	1.0785	0.0722	1.006	0.0516
4.199	0.0761	4.395	0.1055	4.562	0.1338
8.58	0.1297	8.72	0.1801	8.59	0.1805
18.1	0.27	17.65	0.3413	16.775	0.2753

* Initial threshold.

TABLE III. *Showing variation of Fechner fraction* ($\Delta B/B$) *with logarithm of brightness* (*log B*) *in light adaptation.* (*B* expressed in ml.)

L.L.S.		G.K.W.		A.E.O.M.	
log B ml	$\Delta B/B$	log B	$\Delta B/B$	log B	$\Delta B/B$
$\bar{3}.71365$*	0.464	$\bar{3}.69269$*	0.551	$\bar{2}.10117$*	0.255
$\bar{1}.88930$	0.0269	$\bar{1}.91078$	0.0538	0.03023	0.0521
0.65584	0.01366	0.66332	0.0280	0.62133	0.0278
0.94850	0.00756	0.95156	0.0217	0.93523	0.0178
1.27672	0.00805	1.27436	0.0204	1.26352	0.0147
W.L.V.		C.E.B.		P.R.	
log B	$\Delta B/B$	log B	$\Delta B/B$	log B	$\Delta B/B$
$\bar{3}.79195$*	0.600	$\bar{3}.55534$*	0.966	$\bar{3}.71628$*	0.262
0.01927	0.0354	0.03656	0.0652	0.00635	0.0512
0.62701	0.01813	0.64670	0.024	0.66332	0.0293
0.93681	0.01515	0.94386	0.0207	0.95347	0.0204
1.26091	0.01492	1.25049	0.01933	1.22786	0.0164

* Initial threshold.

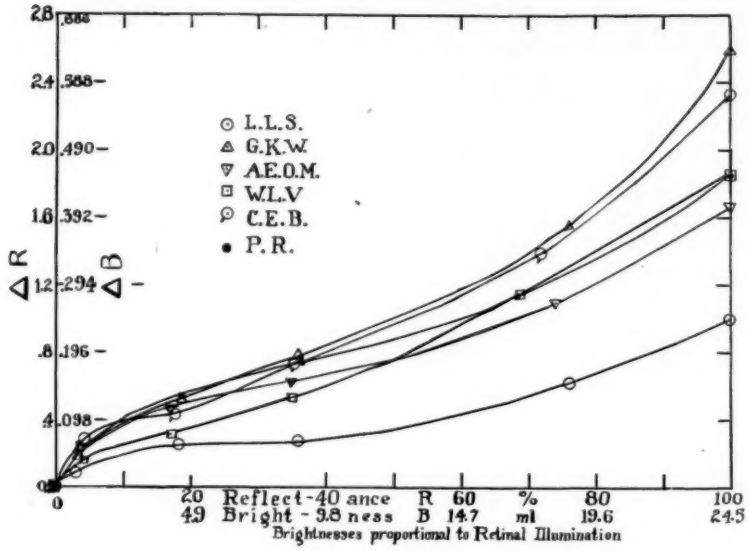

FIG. 6.

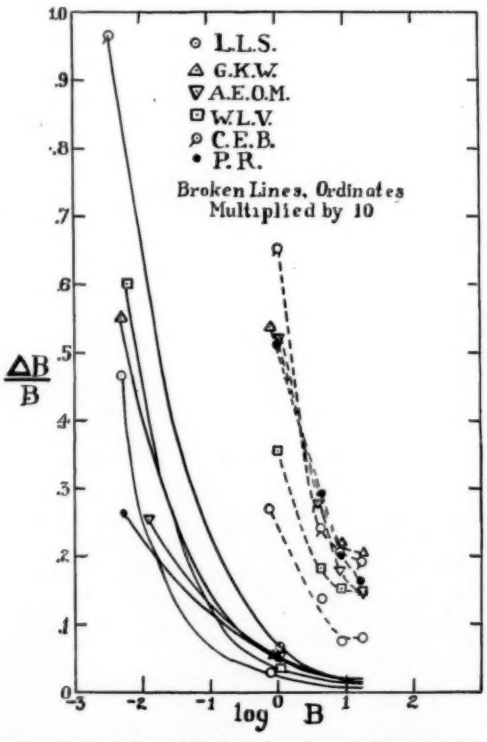

FIG. 7. Variation of Fechner fraction $\Delta B/B$ with brightness B; natural pupil, light adaptation, light surrounding field (4.69 ml).

difference units between these two reflectances The areas under this curve between 0 and 10 percent, 10 and 20 percent, 20 and 30 percent, etc., along the base line were measured by means of a planimeter. By successive additions the total areas from zero to 10 percent, zero to 20 percent, etc., were obtained. These data for one observer are given in Table IV and plotted in Fig. 8. If the areas are expressed in the proper units this curve shows for any reflectances the number of just-noticeable-differences between R and zero,* i.e., between a stimulus which reflects no light and one whose reflection coefficient is R. For example, on this curve the ordinate corresponding to $R=0.50$ is 269; that corresponding to $R=1.00$ is 351. For this particular observer, therefore, there

* Strictly speaking the curve in which $1/\Delta R$ is plotted against R does not extend to $R=0$ but stops at the magnitude of R corresponding to the initial threshold. The areas should therefore be measured not from zero but from R_i, the value of R corresponding to the initial threshold. An additional unit of area should then be added to the results of this measurement, since we know that by decreasing R to some value between 0 and R_i a point can be reached which appears just noticeably darker than the initial threshold. The values of R_i for all the observers are, however, so close to zero (see Table II) that there is no appreciable difference in the results if the simpler procedure is used in which the areas are measured from the zero point on the abscissa.

TABLE IV. *Number of just-noticeable-difference units between various intervals of reflectance for observer L.L.S.* Data from area measurements on $1/\Delta R$, R Curve.

R	J.N.D. units	Total number of J.N.D.'s from $R=0$*
0–5	94.50	94.5
5–10	34.30	128.8
10–15	26.15	154.95
15–20	21.80	176.75
20–25	19.25	196.0
25–30	17.35	213.35
30–35	15.85	229.2
35–40	14.40	243.6
40–45	13.15	256.75
45–50	12.00	268.75
50–55	11.15	279.9
55–60	10.35	290.25
60–65	9.65	299.9
65–70	9.10	309.0
70–75	8.50	317.5
75–80	7.95	325.45
80–85	7.50	332.95
85–90	6.60	339.55
90–95	6.10	345.65
95–100	5.35	351.0

* See starred footnote, page 404.

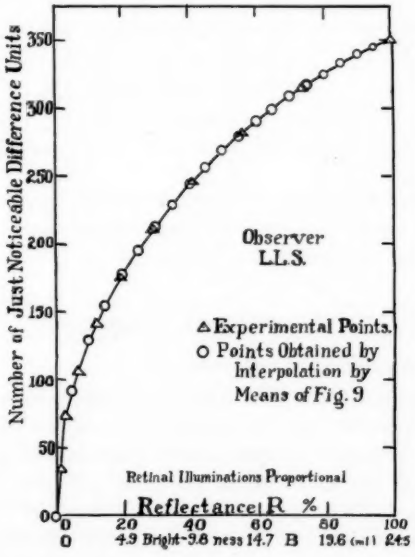

FIG. 8. Showing the number of J.N.D. units corresponding to each reflectance or brightness. Observer L.L.S.

are 269 just-noticeably-different steps between a surface reflecting 50 percent of the incident light and one reflecting no light; between a surface reflecting 100 percent of the light and one reflecting no light there are 351 just noticeable differences. (These results, of course, hold only for the experimental conditions as to illumination, brightness of surrounding field, etc., under which the determinations were made.) If we wish to determine for this observer a series of ten reflecting surfaces between 0 and 100 percent reflectance, we have only to find from the curve the reflectances which correspond to 35.1 J.N.D.'s, 70.2 J.N.D.'s etc. up to 351 J.N.D.'s. The data obtained in this way are given in Table V. The figures in column 2 give in terms of reflectance the specifications for the value scale for this observer.

TABLE V. *Showing series of reflectances dividing area under curve into ten equal parts.* Observer L.L.S.

Area (Number of just-noticeable-difference units)		Reflectance (percent)	
0.	210.6	0.	29.2
35.1	245.7	1.25	40.7
70.2	280.8	3.1	55.7
105.3	315.9	6.5	74.2
140.4	351.0	12.1	100.
175.5		19.6	

In order to obtain a sufficient degree of accuracy in the results of the calculations described above, several precautions are necessary. (1) Because of the peculiar shape of the $(1/\Delta R, R)$ curve and the very rapid drop which takes place at the low reflectances, it is difficult to draw the curve in this region from only a few experimental points. If, however, ΔR is plotted as ordinate and the *square root of* R as abscissa it is comparatively easy to determine the course of the curve through the points plotted. Such a curve for one observer is shown in Fig. 9. By plotting the square root of R instead of R the scale of abscissae is magnified in the region corresponding to the smaller magnitudes of R. The advantage of this is to increase the accuracy with which data may be obtained by interpolation in this region. Data obtained in this way are given in columns 1 and 2 of Table VI. Columns 3 and 4 give R and $1/\Delta R$ calculated from the data of the two preceding columns.

Table VII gives for each observer the total number of just-noticeably-different steps between the two limits of the value scale, i.e., between 0 and 100 percent reflectance or between 0 and

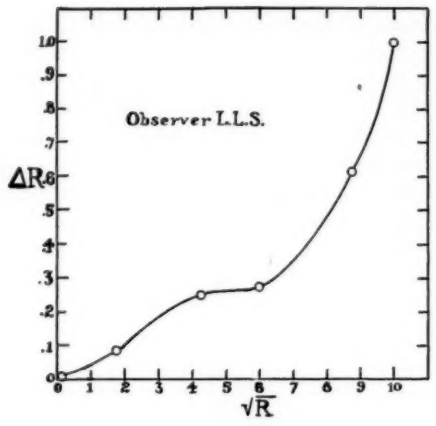

FIG. 9. Showing variation of ΔR with $R^{\frac{1}{2}}$. Curve used in obtaining by interpolation additional data on corresponding amounts of ΔR and R.

TABLE VI. *Data used in plotting curve shown in Fig. 9.* Observer L.L.S.

$R^{\frac{1}{2}}$	ΔR	R	$1/\Delta R$
	Experimentally determined data		
0.1446	0.009685	0.0209	103.2525
1.769	0.08429	3.13	118.64
4.278	0.250	18.3	4.00
5.992	0.2715	35.9	3.6832
8.741	0.615	76.4	1.626
Data obtained by interpolation on ΔR, $R^{\frac{1}{2}}$ curve (Fig. 9)			
0.5	0.0225	0.25	44.444
1.0	0.045	1.0	22.222
1.5	0.070	2.25	14.286
2.0	0.102	4.0	9.8039
3.0	0.185	9.0	5.4054
4.0	0.2415	16.0	4.1408
5.0	0.263	25.0	3.8023
6.0	0.274	36.0	3.6496
7.0	0.350	49.0	2.8571
8.0	0.483	64.0	2.0704
9.0	0.667	81.0	1.4993
10.0	1.0	100.	1.0

TABLE VII. *Total number of just-noticeable-difference units between 0 and 24.5 ml.* (0 and 100 percent reflectance.)

No. of J.N.D.'s	Observer
185	A.E.O.M.
223	W.L.V.
157	C.E.B.
351	L.L.S.
185	G.K.W.
185	P.R.
214	Average

24.5 ml. The differences in the results are probably due in part to actual differences in sensitivity and in part to differences in the criterion used by the observer, i.e., to the degree of certainty required before the two fields compared are judged matched. In spite of the wide variations among the observers in the actual number of just-noticeable-differences corresponding to ten steps on the neutral value scale, however, the value scales themselves for the six observers do *not* differ very markedly from one another. This may be seen from an examination of Table VIII in which the reflectances required to give ten equal value intervals are given for each observer. Curves from these data are plotted in Fig. 10.

TABLE VIII. *Six individual value scales. Showing series of reflectances corresponding to equal value intervals between 0 and 100 percent reflectance. Natural pupil, light adaptations, light surrounding field (19.1 percent), intensity of illumination 22.8 f.c.*

Value	1/	2/	3/	4/	5/	6/	7/	8/	9/	10/
Observer	Percent reflectance									
L.L.S.	1.25	3.1	6.5	12.1	19.6	29.2	40.7	55.7	74.2	100
G.K.W.	1.3	2.5	4.6	9.2	15.7	24.0	35.0	49.5	69.1	100
A.E.O.M.	1.0	2.9	6.4	12.3	20.2	31.0	43.3	57.7	75.5	100
W.L.V.	1.2	3.1	6.4	11.4	17.5	25.7	36.5	50.5	70.5	100
C.E.B.	1.2	3.9	8.0	13.3	20.0	29.0	40.0	55.0	75.0	100
P.R.	0.75	1.9	3.8	8.0	15.0	25.0	37.9	53.4	72.6	100
Average	1.12	2.90	5.95	11.05	18.0	27.3	38.9	53.6	72.8	100

V. VALUE-STEP EXPERIMENTS

1. Description of apparatus

The most essential part of the equipment for this investigation consisted of a series of 60 neutral paper samples whose reflectances were graded by small steps between 2.9 and 88.3 percent. These papers were made especially for this investigation. Their reflectances differed by the following small increments in passing from one to the next in the series: The darkest 15 papers differed on the average by 0.45 percent, the next 15 by an average of 0.97 percent, the next 15 by an average of 1.63 percent, and the lightest 15 papers had an average reflectance increment of 2.31 percent. They were used in

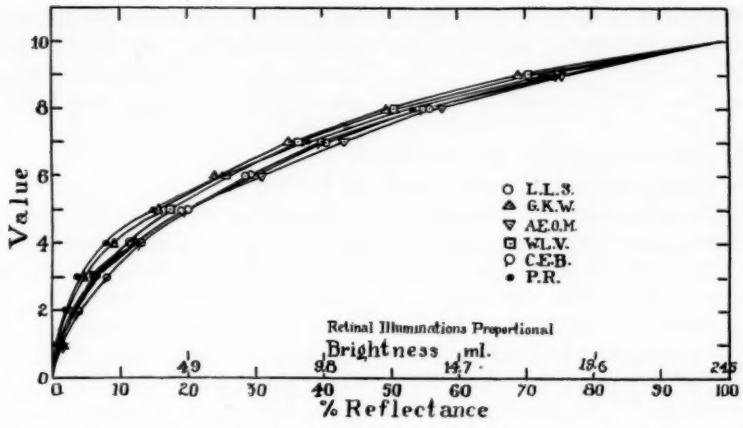

FIG. 10. Showing the relation between value and reflectance, value and brightness, for each of 6 observers.

conjunction with the apparatus shown in Fig. 11. The background, *BB* in this figure, is a piece of cardboard with a matt reflecting surface. Its dimensions were 26 by 35 cm. A series of nine apertures ($A_1 \cdots A_9$) each 1.2 cm square were cut in the cardboard. The apertures were separated

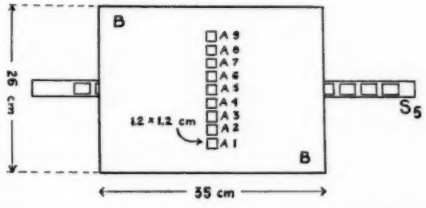

FIG. 11. Apparatus used in determining value scales by the method of value steps. Only one of the nine movable strips (S_5) is shown in this figure.

by a distance of 4 mm. Behind the first and last of this series of apertures, namely A_1 and A_9, were mounted papers of reflectance 2.9 percent and 88.3 percent, respectively. Behind each of the other seven apertures was a movable strip *S*. Fourteen neutral samples selected from the series of 60 papers mentioned above, with the use of as many duplicates cut from the same paper as was necessary, were mounted on the front surface of each strip. By moving the strip past the aperture, the latter could be filled by any one of these samples or by a piece of cardboard similar to that composing the background *BB*. The range of reflectances covered by the samples on each strip was so chosen that it was possible for the observer to select from them a neutral value scale, i.e., a series of papers which differed from one another by equal intervals of sensation. The reflectance of the background was 21.6 percent. The data of the investigation by the just-noticeable-difference method (part IV) were obtained with a background whose reflectance was only slightly lower than this (19.1 percent); consequently a comparison of the results obtained by these two psychophysical methods is made possible. In this investigation the observations were made between the hours of 10 and 4 in a room well lighted by ordinary daylight. Two series of determinations were made, the first in 1926 and the second in 1929–30. In the first series 14 observers were used, 12 making only a single observation and 2 making 2 observations each. In the second series there were 3 observers, two making 4 observations each and the third 2. One observer (I.H.G.) was common to both sets, making 2 determinations in each case. In the first set the intensities of illumination varied between 10 and 35 f.c., and in the second set between 120 and 178 f.c. It was found in an independent series of experiments, described in a later paper, that a variation of illumination within these limits has no appreciable effect on the results, provided the same intensity of illumination falls on the background and on the samples, i.e., provided the

brightness of the background and the samples keep the same ratio to one another with change of illumination.

2. Procedure

The observer first manipulated the middle strip, S_5, until a sample was found which looked (in sensation) halfway between the samples A_1 and A_9, whose reflectances were 2.9 and 88.3 percent, respectively. While the observer was making this first determination, the other six apertures were filled with samples of cardboard similar to that composing the background. When the middle point of the value scale had been selected in this way to fill the aperture A_5 the same procedure was repeated with the aperture A_7; i.e., a sample was selected which appeared midway in sensation between the samples in A_9 and A_5. In a similar manner a sample which appeared midway in sensation between the samples in A_5 and A_1 was selected to fill the aperture A_3. Before proceeding further the observer examined the five samples selected, in order to see if they formed an evenly graded series of sensation intervals. He was allowed to make any modifications of the series necessary to obtain equal sensation intervals. He then proceeded to fill each of the other four apertures in the same way. The final scale was then inspected by the observer and each of the intervals compared in size with every other one. The observer again made any adjustments in the scale which he found necessary. A preliminary investigation showed that the results obtained were independent of minor changes in the distance at which the chart was viewed. Consequently an exact control of this factor was not maintained. The distance from the observer's eye to the chart varied between the approximate limits of 40 and 75 cm. Each of the square apertures, therefore, subtended a visual angle of from one to two degrees.

Of the 14 observers of the first set, 7 had had considerable previous experience in making observations in other psychological experiments in vision. Of the remaining 7, one was an artist and two were physicists. The three observers of the second set were all considerably experienced in this kind of visual judgment. The observers of both sets were, however, entirely unpracticed in making determinations by the method of value steps, except that the one observer common to the two sets had the earlier (1926) experience to aid him in the latter two (1929–30) determinations.

3. Results

The series of reflectances selected by each of the 16 observers are given in Table IX. In order to compare these results with those obtained by the method of just-noticeable-differences, they must be expressed in terms of the same value units. Seven values have been selected which divide the interval between 2.9 and 88.3 percent in the first set (and 3.65 and 82.7 percent in the second set) into equal value intervals. Since in the J.N.D. work reflectances of 2.9 and 88.3 percent (and 3.65 and 82.7 percent) were found to correspond to values of 2/ and 9.63/ (and 2.30/ and 9.42/) respectively, the series of seven values between these limits are as follows: 2.95/, 3.91/, 4.86/, 5.82/, 6.77/, 7.73/, and 8.68/, for the first set (and 3.19/, 4.08/, 4.97/, 5.86/, 6.75/, 7.64/, and 8.53/, for the second set). At the bottom of Table IX are given the average results of the two sets of determinations by the value-step method. In obtaining the average for the first set, equal weights were applied; for the second set the reflectances were weighted by the corresponding number of determinations to obtain the average. For the final average, the first set of reflectances and corresponding values was weighted by the factor 3 and the second set by 2.

The average results of the two methods are shown for comparison in Fig. 12. In this figure the points plotted as circles represent the data obtained by the just-noticeable-difference method; those plotted as triangles, the data obtained by the value-step method. A comparison of the results shows that the scales determined by the two very different methods differ from each other by an average of only 0.064 value steps. This average was taken at the seven points corresponding to the results of the value-step method, the variation at these points being from a difference of one-fiftieth to one-eighth of a value step. The corresponding average reflectance difference at the seven points is 0.62 percent.

The mean variation in reflectance of the

TABLE IX. *Showing series of reflectances selected by each of sixteen observers to form evenly graded scales of values.*

Observer	No. of observations	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
		Percent reflectance								
H. T.	1	2.9	6.4	13.0	18.5	25.3	34.1	46.3	61.6	88.3
M. T.	1	2.9	6.2	9.7	16.6	27.3	35.6	50.8	65.5	88.3
W. C.	1	2.9	6.4	9.7	14.6	22.3	32.3	46.3	61.6	88.3
L. T. L.	1	2.9	5.1	8.8	13.2	25.3	34.1	48.3	65.5	88.3
E. B. T.	1	2.9	4.8	8.8	14.6	22.3	35.6	53.6	70.0	88.3
F. A. C.	1	2.9	6.2	11.7	18.5	25.3	35.0	48.3	64.6	88.3
G. B.	1	2.9	6.2	10.7	17.4	26.4	35.6	50.4	64.6	88.3
C. F.	1	2.9	6.4	10.7	18.5	25.3	34.1	50.4	70.0	88.3
C. K.	1	2.9	5.6	11.7	17.4	25.3	34.1	48.3	64.6	88.3
D. B. J.	1	2.9	6.2	13.2	22.3	32.2	43.5	56.0	74.2	88.3
G. W.	1	2.9	6.7	13.0	17.9	25.3	35.4	52.7	74.4	88.3
L. S.	1	2.9	6.2	10.7	17.4	26.4	35.0	50.8	74.2	88.3
C. E. B.	2	2.9	7.8	15.6	22.3	31.8	43.7	58.5	74.2	88.3
I. H. G.	2	2.9	5.0	10.2	17.9	26.9	38.7	52.4	74.4	88.3
Average of first set (1926)		% 2.90	6.10	11.26	17.66	26.2	36.2	50.9	68.5	88.3%
Value		*2.00*	*2.95*	*3.91*	*4.86*	*5.82*	*6.77*	*7.73*	*8.68*	*9.63*
E. L.	4	3.65	7.7	12.0	16.9	24.0	32.0	47.0	64.6	82.7
B. R.	4	3.65	7.4	12.2	16.6	24.9	32.7	44.5	60.7	82.7
I. H. G.	2	3.65	7.6	12.9	17.5	27.8	37.8	49.3	63.8	82.7
Average of second set (1929–30)		% 3.65	7.57	12.28	16.92	25.1	33.4	46.5	62.9	82.7%
Value		*2.30*	*3.19*	*4.08*	*4.97*	*5.86*	*6.75*	*7.64*	*8.53*	*9.42*
Average of two sets (weighted 3 : 2)		% 3.20	6.69	11.67	17.36	25.8	35.1	49.2	66.3	86.1%
Value		**2.12**	**3.05**	**3.98**	**4.90**	**5.83**	**6.76**	**7.69**	**8.62**	**9.55**

TABLE X. *Showing the mean variations of individual value scales from the average value scales.*

J.N.D. method 6 observers 5 or 6 determinations each			Value-step method 14 observers 1 determination (*) each			Value-step method 3 observers 4 determinations (**) each		
Value	Mean variation In % reflectance	Mean variation In value steps	Value	Mean variation In % reflectance	Mean variation In value steps	Value	Mean variation In % reflectance	Mean variation In value steps
3/	±1.17	*±0.30*	2.95/	±0.55	*±0.13*	3.19/	±1.76	*±0.09*
4/	1.63	*.28*	3.91/	1.52	*.25*	4.08/	1.55	*.10*
5/	1.93	*.25*	4.86/	1.75	*.23*	4.97/	1.88	*.16*
6/	2.42	*.23*	5.82/	1.94	*.18*	5.86/	1.06	*.10*
7/	2.43	*.18*	6.77/	2.47	*.20*	6.75/	.25	*.04*
8/	2.5	*.16*	7.73/	2.64	*.16*	7.64/	.26	*.04*
9/	2.1	*.10*	8.68/	4.51	*.22*	8.53/	.15	*.03*
Average	**2.0**	**0.21**	Average	**2.2**	**0.20**	Average	**1.0**	**0.08**

* Except, two observers, 2 determinations.

** Except, one observer, 2 determinations.

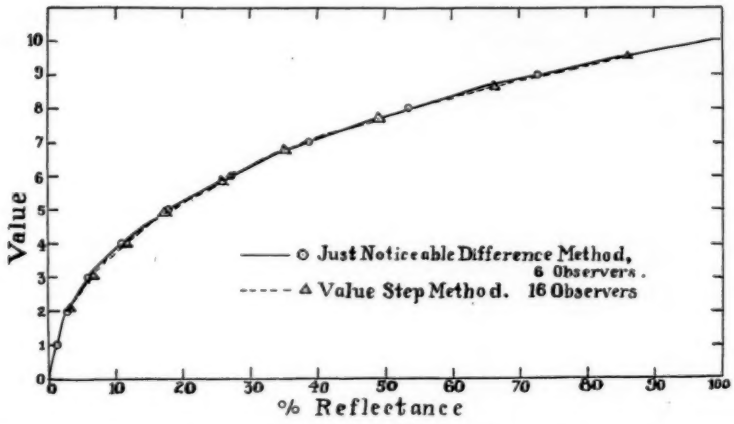

FIG. 12. Comparison of average value scales obtained by the just-noticeable-difference and value-step methods. Gray background, light adaptation.

individual data from the average at each value level have been computed for both sets of experiments. These data are shown in Table X and are plotted in Fig. 13. From this figure (or more exactly by means of a table of interpolated results) the mean variation *in value* can also be estimated. The magnitude of the mean variation is seen to be approximately the same for the two methods, when the variation of 6 observers making 5 or 6 determinations each by the J.N.D. method is compared with the variation of 14 observers making a single determination by the value-step method. But in the case of the last set in which two observers made 4 and another made 2 determinations, by the value-step method, the variation is less than one-half as much. The precision with which an average value scale can be determined by 14 unpracticed observers each making *only a single set of determinations by the method of equal value steps is therefore seen to compare favorably with the precision attained in the results of 6 observers making 5 to 6 separate determinations by the just-noticeable difference-method;* and with an increased number of determinations, the precision of the former method can be made considerably better. This conclusion is of especial interest in view of the fact that the method of equal sense differences, of which the value-step method is a particular case, has often been thought to give results of low precision, and to be a difficult method for use with unpracticed observers. It is undoubtedly difficult to select the midpoint in sensation between two sensations which differ greatly. If, however, this sensation interval is divided and subdivided into a number of smaller sensation intervals, each of which can be compared with every other, the judgment becomes far easier. The results of the present investigation show that when such a procedure is used, and when the experimental conditions are properly controlled, the value-step method is entirely feasible.

VI. Summary and Conclusions

(1) It has been found by the just-noticeable-difference method that a series of neutral reflecting surfaces whose reflectances are 1.12, 2.90, 5.95, 11.05, 18.0, 27.3, 38.9, 53.6, 72.8, and 100.0 percent, when placed on a background whose reflectance is about 18 percent and illuminated by 22.8 f.c. of noon sunlight (5200°K) form for the average observer a series of equal differences in value.

(2) It has been found by an adaptation of the direct equal-sense-difference method, for a background of about 18 percent reflectance, for illuminations varying over a large range of intensities (within the limits of ordinary indoor illumination), and a quality equivalent to that of skylight, that results practically identical with the results stated in item (1) are obtained.

(3) The value scales determined by the two very different methods described in this paper

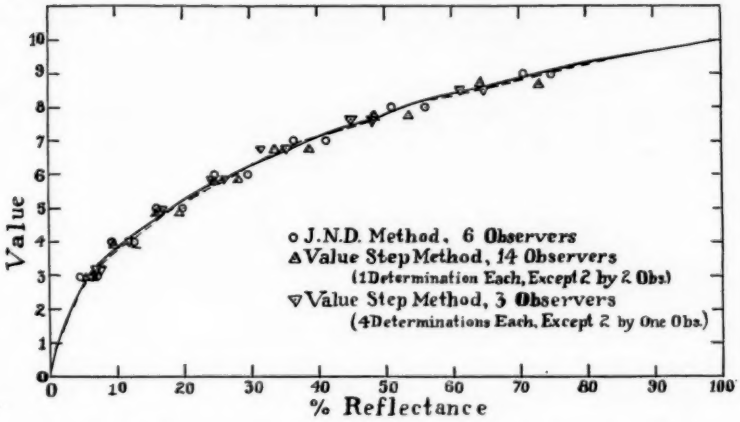
Value
% Reflectance
o J.N.D. Method, 6 Observers
Δ Value Step Method, 14 Observers
(1 Determination Each, Except 2 by 2 Obs)
▽ Value Step Method, 3 Observers
(4 Determinations Each, Except 2 by One Obs.)

FIG. 13. Showing mean variation of individual value scales from the average.

differ, for the average of 6 observers in the first case, and for the weighted average of 16 observers in the second case, on the average by only 0.6 percent in reflectance or about 0.06 of a value step (where a value step is as defined above), i.e., by 1/150th the sensation difference between black and white.

(4) The precision with which an average value scale can be determined by 14 unpracticed observers each making *only a single set of determinations* (2 observers made 2 determinations) by the method of equal value steps is equal to that attained in the results of 6 observers each making 5 to 6 separate determinations by the just-noticeable-difference method; and the precision of the former method for 3 observers for the average of 4 determinations or less is more than twice as good.